Engagement, Disengagement and Exit

Elizabeth Maggie Penn[*]

November 13, 2013

Abstract

This paper considers the possibility of information transmission across groups when any group may choose to unilaterally exit society. My main concern is the tradeoff social groups face between the informational benefits of associating with other groups in a large society versus the costs imposed by preference diversity on receiving their preferred outcome. When these costs are sufficiently high, groups may prefer exit to association. The results of this paper characterize the associations of groups that can be sustained in equilibrium and, within those associations, the types of groups that choose to engage in or disengage from the process of inter-group communication. The results demonstrate that there can be benefits or costs associated with the inclusion of preference extremists in a diverse society, whether or not those groups choose to actively communicate with outgroups. The results also speak to both institutional and intra-group mechanisms for fostering communication across groups.

1 Introduction

Democratic theorists have long recognized the value of information to a well-functioning society. Information is required for a population to critically evaluate its leaders, to confront conflicts that arise, and, more generally, to secure its well-being through its own agency. In this vein, John Adams writes that "...wherever a general knowledge and sensibility have prevailed among the people, arbitrary government and every kind of oppression have lessened and disappeared in proportion."[1] This view suggests that information aids democracy because it enables individuals to reflect upon their own situations and empowers them to take action; it serves as a check against leaders pursuing undemocratic goals. A different view of the role of information in democratic society is that information *per se* constitutes the source of a democracy's value. In documenting the history of Athenian governance, Ober argues that the participatory nature of Athenian democracy, along with smart institutional design, served to consolidate information that was widely dispersed throughout the population, and that this aggregation and distribution of knowledge played a causal role in Athens' success relative to its peer polities. The idea that democratic institutions serve as systems for "...organizing what is known by many disparate people"[2] has arisen in many manifestations, in both ancient and modern work.[3]

If we take seriously the idea that the dissemination of information is essential to a democratic society, and that with a large and diverse population this information may be both dispersed and privately held, then we must also consider the possibility that people may or may not choose to share what they know with others. A large literature has been devoted to the study of strategic information transmission, with a key and robust finding being that as the preferences of individuals become increasingly divergent, their ability to credibly communicate with one another is reduced. This phenomenon is exacerbated in settings in which communication between individuals is both

---

[1] Adams (1765).

[2] Ober (2008, p. 2).

[3] Arguments drawing upon the notion of "wisdom of crowds" can be found in work by Condorcet (1785), Hayek (1945), Surowiecki (2005) and Page (2008), among many others. I will return to the Athenian case later in this paper.

costless and non-verifiable.[4] However, it is precisely this kind of setting that might characterize a democratic society in which information is highly decentralized, there are no barriers to whom one may associate and communicate with, and people may freely say what they like without fear of punishment.

I begin this paper by presuming that information is valuable to everyone for decision making, and by acknowledging the fact that diversity, free association and free speech are the hallmarks of many democratic societies. Phrased differently, much of communication in a free society is cheap talk. Using this observation as a starting point, I am interested in the possibility of information transmission across diverse social groups that are internally homogenous, and when it is (or is not) in the interest of a group to co-exist with other groups. My main concern is the tradeoff that groups face between the informational benefits of associating with others in a large society versus the policy costs of that association; if, for example, several distinct groups choose to associate with each other, each must bear a negative externality stemming from their divergent preferences over outcomes. If these costs are sufficiently high, one or more groups may prefer unilateral exit to association.[5] In a well-functioning society, the sharing of useful knowledge and ideas incentivizes groups to associate with each other despite their differences.

The results of this paper characterize the associations of groups that can be sustained in equilibrium and, within those associations, the types of groups that choose to engage in or disengage from the process of inter-group communication. These "associations" are precisely the collections of groups for which some information transmission between groups is possible, and for which the benefits of this information outweigh the costs stemming from preference diversity within the association. In equilibrium the choice of a group to "associate" necessarily represents an improvement in expected utility relative to the choice to "exit." As a group can always choose exit, members of an equilibrium association are always made weakly better off by their decision to associate.

---

[4]Crawford and Sobel (1982).

[5]Several recent papers address questions concerning inter-group interactions in heterogeneous societies. See, for example, Eguia (2012), who examines incentives for assimilation, and Schnakenberg (2013), who looks at the relationship between group identity and symbolic behavior.

Consequently, while larger associations may not maximize social welfare in a utilitarian sense, in equilibrium they virtually always improve the well-being of some group relative to its exit option while maintaining the well-being of all other members of the association above minimum levels. Thus, while not precisely Rawlsian, larger associations can be considered normatively desirable in this sense.

To briefly and informally describe the dynamics underlying the model, consider a country composed of two distinct groups: a coastal group and an inland group. Suppose this country faces the threat of foreign invasion, and must decide how best to thwart an attack. Each group is uncertain about the particular strategy that will be employed by an enemy, but each also has some private information concerning the country's vulnerability to an attack by land or by sea. Each group independently decides upon a division of its military's resources between ships and cavalry. All else equal, the coastal group would prefer that more resources be dedicated to ships, because ships are better able to protect their direct interests; similarly the inland group would prefer more resources be dedicated to cavalry.

Suppose that these groups can costlessly communicate to each other what they know about the likelihood of an invasion by land or by sea. If communication is truthful and credible then it will provide both groups with valuable information that will enable everyone to arrive at a more accurate decision prior to making the costly choice of resource allocation. Of course, the tradeoff the groups face is that with this larger pool of information comes a larger set of potential biases informing the decisions of others. The coastal group may like having better information about the country's vulnerability to attack, but may dislike how the inland group chooses to utilize its own information, and in particular, may wish that the inland group dedicated more resources to preventing an attack by sea. If this tradeoff becomes too great, the groups may choose to part ways; they may choose to forego the others' information (and the implicit commitment to aid in defending each other's territory) in favor of no longer being subject to the choices made by the other.

In the story just told, a group may choose to forego the informational benefits of communication

with outgroups in favor of being more able to target outcomes to its own biases. In this case, the group chooses *exit*. In the "ships versus cavalry" example, a coastal group's choice of exit could be reflected in the group's decision to no longer consent to pooling military resources with the inland group, because the inland group's military is not capable enough of defending an attack by sea. Alternatively, the group may choose to engage in a process of communication with outgroups but not reveal any meaningful information to those groups. This might occur, for example, if the coastal group receives private information that perhaps the optimal tradeoff between ships and cavalry favors cavalry more than previously thought. In this case the coastal group might choose to withhold that information from the inland group to prevent them from biasing outcomes even farther toward cavalry. At the same time, pooling military resources with the inlanders may still be optimal for the coastal group as it enables them to reap the informational benefits of the (truthful) messages of the inlanders. I refer to a group's choice to associate with others but conceal its information from them as *disengagement*. Last, the group may choose to associate with others and to truthfully reveal its information to all. Such groups benefit from truth-telling because their honesty better informs the choices of others, and the choices of others directly affect everyone in the association. I refer to these groups as *engaged.*

1.1 Related literature

In recent years a large formal-theoretic literature has arisen on the topic of communication as a mechanism for democratic policy-making. By and large, this literature models deliberative democracy as verbal (cheap talk) communication between privately-informed participants seeking to arrive at a collective choice.[6] The widespread assumption in this literature that deliberation's value lies in its ability to successfully aggregate privately held information has been criticized by some as

---

[6]See Austen-Smith and Feddersen (2006), Gerardi and Yariv (2007), Meirowitz (2007). Notable departures from the cheap talk framework are Hafer and Landa (2007) and Dickson et al. (2008), which model deliberation as "self discovery" in which the validity of a message is not its inherent truth, but whether it successfully activates previously held, latent beliefs.

an incomplete, and perhaps wholly incorrect, way of thinking about political discourse. Its critics argue that such an account of the deliberative process omits key, philosophical reasons for deliberation, such as a desire on the part of participants to publicly articulate reasons for their desired outcomes.[7] At the same time, Landa and Meirowitz (2009) note that if deliberation is to serve a purpose beyond allowing participants to simply coordinate on a particular policy choice—if the purpose is to meaningfully change the preferences of participants over outcomes—then participants must face some uncertainty about some aspect of policy choice. If this is the case then the cheap talk assumption is conservative, as it represents the most challenging setting in which to study the incentives for truthful communication. To quote from Landa and Meirowitz's overview of this literature, and their reflections on some of the deeper methodological issues surrounding game theoretic versus normative theoretic approaches to the study of deliberation,

> "[T]he impulse behind the game-theoretic analysis of deliberation is to 'earn' the sincerity by reconstructing it as equilibrium behavior rather than assuming it by default... The value of doing so is not only explanatory. Unless we understand the conditions under which the incentives in deliberative environments encourage agents to be sincere or fully revealing, as opposed to insincere or withholding information, we cannot hope to offer a coherent (stable) normative argument for institutional design."

The model presented in this paper bears strong connections to this prior work in that I am similarly interested in how institutional arrangements can incentivize individuals to productively communicate with one another. However, two features of the model distinguish it from much of the work on deliberative democracy. The first concerns the process of collective choice while the second concerns the "opt in" nature of the deliberative body.

Austen-Smith and Feddersen (2005) make a distinction between *deliberation* and *debate*. In the former, two or more privately informed agents engage in cheap talk prior to arriving at a collective decision *via a voting rule*; in the latter, the cheap talk communication simply precedes *some decision being made*. This project would fall into the latter category, in that I model outcomes

---

[7]See Minozzi et al. (2013).

6

as a function of decentralized decision-making on the part of the participants; in other words, once communication has occurred, participants face no further strategic considerations. However the larger point distinguishing this work from much of the literature on deliberation is that this literature takes the deliberative body as static or as an institutional lever. I am interested in the endogenous formation of associations that arise naturally because of the benefits of communication, and the varying incentives faced by groups within such associations. In this model, universally beneficial deliberation (or perhaps more accurately, "debate") is a prerequisite for the existence of societal stability; in its absence, one or more groups will always be incentivized to exit. This idea is related to the argument that if institutions fail to foster beneficial communication between political actors, then those institutions have themselves failed, and may be perceived as illegitimate.[8]

In considering the endogenous formation of associations for which membership is mutually beneficial for the participants, this paper is related to a large body of work on endogenous group formation, network formation and political confederation. Baccara and Yariv (2013) develop a model of endogenous peer group formation in which people differ in their relative preferences for two public projects and each person chooses both a peer group and a public project to contribute to. This paper is most related to an extension of their baseline model to a finite type space, in which a collection of (endogenously formed) peer groups is *stable* if no agent wants to leave their group given the tastes of others, and foreseeing the contributions that others will make. Unlike this paper, their model contains no private information and, dependent on a choice of contribution, each agent's decision factors into her group members' preferences in an identical way. Moreover, they allow for a rich collection of groups to form endogenously. In contrast, this paper considers a single association as a "group of groups" in which members may differ in both the quality of their information and the importance of their choice relative to the choices of others. By limiting groups to simply decide between "association" or "exit" my focus is on a group's choice to participate in either a private sphere or a public one, and, in the latter case, the group's incentives for truthful communication dependent on the behavior of the other groups that have similarly chosen

---

[8]See Cohen (1989) and Manin et al. (1987).

participation in the public sphere.

The "association / exit" choice faced by the groups in this paper is closely related to Cremer and Palfrey's (1999) model of political confederation. Similar to this paper's notion of an association as a group of groups, those authors model a confederation as a collection of states, or smaller political units. Voters have preferences over two dimensions of a possible "constitution," with the first dimension capturing a degree of centralization of the political system and the second capturing a representation scheme. The authors focus on the existence (or lack thereof) of a majority rule equilibrium over this two-dimensional space, with voters facing a tradeoff between the riskier outcomes associated with decentralized government versus the higher policy costs incurred under centralized government. The tradeoff between the better decisions that a larger decision-making body is capable of versus the increased preference heterogeneity informing those decisions is precisely the tradeoff captured in the association / exit decisions faced by groups in my model. Moreover, the representation dimension considered by those authors is directly analogous to the varying policy (or "influence") weights that can be assigned to groups in my model. Those similarities aside, their model focuses on collective choice over confederation type, while this model focuses on unilateral decisions by groups to join or leave an association, and communication incentives within the ensuing association.

Finally, the inter-group communication subgame of this model utilizes a messaging technology similar to one developed by Galeotti et al. (2013), who study the endogenous formation of "truthful networks" when agents are constrained in the audiences they can speak to.[9] The (directed) network is generated by the equilibrium communication strategies of the players; a link from i to j represents an incentive on i's part to communicate truthfully with j. One distinction between this model and theirs is that here the audience a group can speak to is endogenously determined by the

---

[9]A number of recent works have employed this framework to address various topics in political economy. Dewan and Squintani (2012) study endogenous factions; Dewan et al. (2011) look at the question of optimal executive structure; Patty (2013) focuses on the inclusion and exclusion of agents in a deliberative body; Patty and Penn (2013) study sequential decision-making in "small networks" when communication is costly; and Gailmard and Patty (2013) look at questions of delegation.

association / exit decisions of the groups themselves. Another difference is that the groups in my model can be differentiated in part by the quality of the information they have and their influence on outcomes for the association, whereas agents in their model are homogeneous in these respects. Like Galeotti et al., I focus for the most part on communication strategies that are either truthful or uninformative. Thus, the communication strategies in this paper similarly describe a truthful network of sorts, in which a link from group i to j exists if a stable association containing i and j exists, with i being fully incentivized to truthfully communicate to that association. As in Galeotti et al., other equilibria to the model do exist in which communication between groups can be partially informative. These equilibria are discussed (and, in an example in Section 6, calculated) later in the paper.

The paper proceeds as follows. Section 2 establishes the model of group association and cheap talk. Section 3 presents several general results, including a characterization of the conditions required for truthful messaging and for voluntary association, a partial ranking of equilibria on the basis of social welfare, and a more detailed discussion of the two-group case. Section 4 works through two examples to provide intuition for the kinds of comparative statics that emerge from the model. Both examples focus on how equilibria vary as the bias of a third group changes from centrist to extreme. Section 5 discusses how allocating policy discretion across a collection of groups can be used as an instrument to induce better communication and grow the size of an association. Section 6 discusses how, by foregoing some of its own information, a group can improve its ability to communicate with another group. This strategy is compared with a different mechanism for improving communication, namely obfuscation of a group's own information in the form of a semi-separating strategy. Section 7 presents a different intra-group mechanism for improving communication with an outgroup: disaggregation of the group's own information by limiting within-group communication. Section 8 steps back from the mechanics of the model to discuss some connections between this project and more general questions of institutional design and democratic choice. Section 9 concludes.

2 The Model

I consider a society consisting of a collection of groups $g \in G$, with $|G| = n$, and with each group consisting of n_g individuals ("members of g"). Every person belongs to a single group. Groups are differentiated solely on the basis of their respective sizes and the preferences of their members. These preferences are represented by a vector of biases, $\beta = (\beta_1, ..., \beta_n)$, which are common knowledge, as are the number of individuals in each group, n_g. An individual's bias will affect his payoff from both his own activities and the activities undertaken by the individuals he has chosen to associate with. Every individual undertakes an activity $y \in \mathbb{R}$, and a member of g's payoff to a particular activity y is:

$$u_g(y, \theta; \beta_g) = -(y - \theta - \beta_g)^2,$$

where $\theta \in \Theta$ represents a "state of the world" drawn from a Uniform$[0, 1]$ distribution over state space $\Theta = [0, 1]$. Upon realization of θ, every person in the population receives a conditionally independent, private signal $s_i \in \{0, 1\}$, according to the probability mass function:

$$\Pr[s_i = x | \theta] = \begin{cases} 1 - \theta & \text{if } x = 0 \\ \theta & \text{if } x = 1. \end{cases} \tag{1}$$

Let $s_g = \sum_{i \in g} s_i$ be the number of positive signals received by the members of group g. Thus, a member of g considers an "ideal action" to be $\theta + \beta_g$, and prefers activities to be as close to this action as possible.

2.1 Action and communication

In the following sections I describe the three types of decisions that each group can make: an *association* decision, a *message* and an *activity*. For reasons that will become clear later, I begin by presuming that group members will truthfully communicate their private signals, s_i, to one another so that I may conceive of "the group" as a unitary actor that will receive n_g total signals. This is because I am interested in the tradeoffs groups face when choosing whether to exit or

enter society at large, and how both the size of a group and its relative bias affect this decision.[10] While I assume that individuals will always communicate truthfully with members of their own group, groups may choose to associate with "society at large" *prior to realizing their own signals*. A choice to associate with the larger society implies that the group agrees to both participate in a process of public communication with all other groups that have similarly chosen to associate with society at large, and to receive payoffs that are dependent on the actions of the groups in the association.

As noted above, each group faces a tradeoff when choosing whether or not to associate with society at large. On the one hand, members of the group could choose to disassociate from the whole, and to receive a payoff that is dependent on only the activities of the group members and the information that that subset of individuals could provide. On the other, if the group chooses to associate with society at large then members can, potentially, reap the informational benefits that the larger pool of signals may afford. However, association comes at the cost of incurring a payoff that dependent on the actions of all members of the association. Thus, each member of an association incurs a negative externality associated with the actions of others who have similarly chosen to associate, but who hold different biases.

Leaving aside the role of communication for the moment, social outcomes are captured via a decision that every group g makes over its choice of an activity, $y_g \in \mathbb{R}$ with $y = (y_1, ..., y_n)$. If a group chooses association its members receive a payoff that is dependent on the activities of all members of the association. In this payoff, the activity of group h in association R is weighted by the exogenous term α_h^R by each member of the association. For any $R \subseteq G$, α_h^R captures the relative influence of h's decision within the association. These α terms could, for example, be proportional to the population of each group within the association, proportional the wealth of the

---

[10]Clearly, larger groups will have better information about θ than smaller groups, and a later section of this paper considers whether members of a large group can be made better off by committing to limit their own within-group communication. Doing so enables a large group to more credibly commit to having less information about θ. As I will discuss later, this commitment may induce other groups to regard information communicated by that group as truthful.

groups, or something else entirely. It follows that the payoff to group g from association with R is

$$u_g^{\mathtt{a}}(y, R|\theta) = -\sum_{h \in R} \alpha_h^R (y_h - \theta - \beta_g)^2. \tag{2}$$

If group g chooses exit then its payoff is solely dependent on its own choice of activity, y_g so that

$$u_g^{\mathtt{x}}(y|\theta) = -(y_g - \theta - \beta_g)^2. \tag{3}$$

The above payoffs to association and exit reflect the fact that, with equal information about state of nature θ, a group would always choose exit over association, as exit enables the group to perfectly target its activity to its bias and avoid the externalities associated with preference diversity. However, a group that chooses exit cannot receive information from any other group, and so updates its estimate of θ solely on the basis of the number of positive signals received by its members, s_g. Conversely, members of an association may receive information from the groups that have similarly chosen association, in the form of a public, cheap talk message that each group sends to the association concerning the number of positive signals its members have received. If informative, these messages will improve each association member's estimate of θ. Thus, groups choosing association also choose a message to be conveyed to the other groups that have chosen association.

To summarize, groups make three types of choices. First, each group makes an association decision $a_g \in \{\mathtt{a}, \mathtt{x}\}$ denoting association or exit, respectively. Second, if choosing association, each group picks a message to be sent to the other groups in the association, $m_g \in \{0, ..., n_g\}$, communicating the sum of positive signals claimed to have been received by the group. Last, each group chooses an activity, y_g.

2.2 Messaging Equilibria and Societal Stability

I am interested in characterizing the types of associations that can form, and group behavior within these associations, when communication and association decisions are strategic. My focus is on pure strategy perfect Bayesian Nash equilibria. Groups' actions are decomposed into two parts,

which consist of an association decision and a messaging strategy. A (pure) association decision is simply a choice $a_g \in \{\texttt{a}, \texttt{x}\}$. If $a_g = \texttt{a}$, a messaging strategy ρ_g maps a sum of signals received by group g and a set of associators, $R \subseteq G$, into a public message to the association, m_g. Thus, $\rho_g : \{0, ..., n_g\} \times 2^{|G|} \to \{0, 1, 2, ..., n_g\}$.

The signaling technology defined in Equation 1 implies that a player's posterior belief about θ after observing m trials (signals) and k successes (observations of $s = 1$) is characterized by a $\text{Beta}(k + 1, m - k + 1)$ distribution, which implies:

$$
\begin{aligned}
E(\theta|k, m) &= \frac{k + 1}{m + 2}, \text{ and} \\
V(\theta|k, m) &= \frac{(k + 1)(m - k + 1)}{(m + 2)^2(m + 3)}.
\end{aligned}
\tag{4}
$$

It follows that if group g observes the truthful revelation of k successes and $m - k$ failures, then it is always optimal for g to select:

$$
y_g^*(k, m) = \frac{k + 1}{m + 2} + \beta_g.
\tag{5}
$$

For the sake of parsimony, and to clarify the arguments I wish to make, I focus on communication strategies within the messaging subgame for the association that are either separating or pooling.[11] This focus simplifies the analysis by enabling us to consider only three actions taken by groups in equilibrium, with these actions implicitly capturing the group's association decision, messaging strategy and policy choice. Groups can either exit, associate and communicate truthfully, or associate and babble. The equilibria defined below can therefore be characterized by

---

[11]The focus on communication strategies that can take one of two forms (truthful or uninformative) reduces to a focus on pure strategies when players have only two signals. Thus, while analogous to the analysis in Galeotti et al., my focus on separating equilibria is a stronger restriction than pure strategies are in their framework. The additional restrictiveness stems from the larger message space considered here. As those authors note, the existence of Pareto-improving mixed strategies in their setting is possible, as it is here. Moreover, this model also yields semi-separating pure strategy equilibria, which are discussed in more detail through an example I construct in Section 6. The focus on separating equilibria greatly simplifies the analysis while capturing qualitative features of the model that would similarly be found by expanding my scope to consider these different types of equilibria.

considering divisions of G into three possibly empty and mutually disjoint sets: E, D, and X, where $E \cup D \cup X = G$. These sets correspond, respectively, to the groups that choose $a_g = $ a and $\rho_g(s_g, R) = s_g$ for all $s_g \in \{0, ..., n_g\}$; the groups that choose $a_g = $ a and $\rho_g(s_g, R) = 0$ for all $s_g \in \{0, ..., n_g\}$; and the groups that choose $a_g = $ x. I denote the collection of E, D, X divisions of G by $\mathcal{S}$ with element ("society") $\sigma \in \mathcal{S}$.

I refer to groups in E as those that have chosen to engage, as these groups will both associate with society at large and truthfully reveal their information to the other groups that have chosen association. I refer to groups in D as those that have chosen to disengage, as these groups will associate with society at large but reveal no information to other groups. Groups in X have chosen to exit society, receiving no information from outgroups and taking actions informed solely by the information provided by their own members. The set of groups $R = E \cup D$ is termed the *association*, as these groups have chosen to engage in a public messaging game. Let $n_E = \sum_{g \in E} n_g$ and $s_E = \sum_{g \in E} s_g$ represent, respectively, the number of individual group members in E and the sum of positive signals received by those members. Sequential rationality implies that actions, y_g, maximize groups' expected payoffs given their own signals and the messages they receive. Thus, for a division of groups $\sigma \in \mathcal{S}$ with $\sigma = \{E, D, X\}$, let $y_\sigma = (y_{g,\sigma})_{g \in G}$ denote a sequentially rational profile of actions for society σ, so that:

- For $g \in E$, $y_{g,\sigma} = y_g^*(s_E, n_E)$,

- For $g \in D$, $y_{g,\sigma} = y_g^*(s_E + s_g, n_E + n_g)$,

- For $g \in X$, $y_{g,\sigma} = y_g^*(s_g, n_g)$.

Before defining the notion of societal stability used for the remainder of the paper, I define a set of *messaging equilibria* for each possible association $R \subseteq G$ that satisfy the equilibrium refinements discussed above.

Definition 1 *For any $R \subseteq G$, a division of R into two disjoint subsets, $\{E, D\}$ with $E \cup D = R$, is a **messaging equilibrium** for R if the following three conditions are met:*

14

1. Individuals have equilibrium beliefs. For all groups in association R, public messages sent by groups $g \in E$ are taken as equal to s_g and public messages sent by groups in $g \in D$ are disregarded as uninformative.

2. Actions are sequentially rational given groups' own signals and the messages they receive.

3. Groups that truthfully message have no incentive to lie. This means that for all $g \in E$, messaging strategy $\rho_g(s_g, R) = s_g$ for all $s_g \in \{0, 1, ..., n_g\}$ offers g a weakly higher expected payoff than any other strategy that could be taken by g, given the (correct) beliefs, actions and equilibrium messaging strategies of the other groups in association R.

Using the above definition of a messaging equilibrium for R, let

$$\mu(R) = \{\{E, D\} : \{E, D\} \text{ is a messaging equilibrium for } R\}.$$

Thus, $\mu(R)$ is the set of all messaging equilibria for association R. Last, let $EU_g(y_\sigma)$ be the expected utility of group g after messaging, association and (implicit) activity choices have occurred as dictated by σ. We are now in a position to define a notion of societal stability.

Definition 2 *An $\sigma \in \mathcal{S}$ with $\sigma = \{E, D, X\}$ is **stable** if the following three conditions are met:*

1. The set $\{E, D\}$ constitutes a messaging equilibrium for $R = E \cup D$. Thus, $\{E, D\} \in \mu(E \cup D)$.

2. For $g \in X$ actions are sequentially rational given the groups' own signals.

3. Groups have no desire to change their association decisions.

   - For all $g \in R$, $EU_g(y_\sigma) \geq EU_g(y_{\sigma'})$,
     where $\sigma' = \{E \setminus \{g\}, D \setminus \{g\}, X \cup \{g\}\}$.

   - For all $g \in X$, $EU_g(y_\sigma) \geq \max\limits_{\{E', D'\} \in \mu(R \cup \{g\})} EU_g(y_{\sigma'})$,
     where $\sigma' = \{E', D', G \setminus E' \cup D'\}$.

15

The final stability condition deserves particular attention. This condition states that no group can strictly benefit by changing its association decision. For groups in association R the condition is straightforward, because it implies that they prefer remaining in R, given the current messaging equilibrium $\{E, D\}$, to exit. For groups in X the association decision poses a potential ambiguity, because if $g \in X$ chooses to enter association R the messaging strategies of the groups in R may change. Thus, a group g that has chosen exit must compare the expected utility of exit to the expected utility of being in association $R \cup \{g\}$ *given a new messaging equilibrium that his entry will generate*. As $\mu(R \cup \{g\})$ may not be single-valued, this calculation involves g choosing an element of $\mu(R \cup \{g\})$ to evaluate potential benefits of entry with respect to. In this case I assume that g makes its calculation using a messaging equilibrium associated with association $R \cup \{g\}$ that maximizes the expected utility of association for g. While this assumption seems strong, I discuss in Section 3.1 that if a messaging equilibrium for association R maximizes the expected utility of some $g \in R$ then it maximizes the expected utility of *every* $g \in R$; in other words, groups in an association have the same preferences over messaging equilibria. This stems from the fact that the only gain to association is a reduction in the residual variance of each group's estimate of θ due to information transmission. Due to the "shared policy-making" nature of group utility functions, each group in the association benefits equally from this reduction in variance. Thus, g evaluates its entry into R by considering an equilibrium that is utility-maximizing for all members of R, including itself.

3 General Results

I begin this section by deriving conditions for truthful communication by group g to association R, and by deriving the conditions that characterize groups' association decisions. Together, the conditions can be used to characterize instances of societal stability. The second part of this section presents several welfare implications of the model and the final part of this section considers these conditions for the special case of a society composed of two groups. I begin with the following

lemma, which shows that the incentive for a group to truthfully communicate to an association R is most difficult to satisfy when that group seeks to misstate the signal of a single one of its members. Thus, the lemma shows that if it is profitable for g to misstate the signals of some of its members then it is profitable for g to misstate the signal of a single one of its members. The lemma is used to simplify the condition for truthful communication by enabling us to only consider instances in which it is profitable to misstate a single signal. All proofs are in the Appendix.

Lemma 1 *Let g have n_g members who have received a total of $s_g \leq n_g$ positive signals. If revealing $\tilde{s} \neq s_g$ positive signals represents a profitable lie for g, then one of two conditions holds. If $\tilde{s} > s_g$ then claiming $s_g + 1$ positive signals is also a profitable lie. If $\tilde{s} < s_g$ then claiming $s_g - 1$ positive signals is also a profitable lie.*

Lemma 1 states that if Group g has an incentive to communicate dishonestly then g also has an incentive to over- or under-report s_g by a single signal. This result enables us to derive the following condition for truthful public communication from Group g to association R. Of course, the condition must take into account the fact that some groups in R will themselves communicate truthfully while others will not. The existence of both of these types of groups within the association affects the incentives for g to communicate truthfully. On the one hand, the existence of babblers within R means that fewer credible signals are revealed to others within the association. This increases the "manipulative impact" of a false signal by g, and thus lowers g's incentive to reveal a false signal by potentially increasing the cost of such a signal. On the other hand, a false signal by g has a different, lower, manipulative impact on the policy choice of a babbling group than it does on the policy choice of a truthful group. This is because groups that babble have strictly more information than groups that are truthful: they have their own, private, information in addition to the information provided by the groups in E. For the condition below, let $E_{-g} = E \setminus \{g\}$. In the same fashion, let $s_{E_{-g}} = \sum_{h \in E_{-g}} s_h$, the sum of positive signals received by the groups in E_{-g}, and let $n_{E_{-g}}$ be the total number of signals received by the groups in E_{-g}.

Condition 1 *Let R be an association, with $E \subseteq R$ being the groups that truthfully reveal their signals and $D \subseteq R$ being those that babble.* **The truthful messaging condition** *for group $g \in E$ is then:*

$$\sum_{h \in E_{-g}} \frac{\alpha_h^R}{2(n_{E_{-g}} + n_g + 2)^2} + \sum_{j \in D} \frac{\alpha_j^R}{2(n_{E_{-g}} + n_j + n_g + 2)^2} \geq$$

$$\left| \sum_{h \in E_{-g}} \left(\frac{\alpha_h^R}{n_{E_{-g}} + n_g + 2} \right)(\beta_h - \beta_g) + \sum_{j \in D} \left(\frac{\alpha_j^R}{n_{E_{-g}} + n_j + n_g + 2} \right)(\beta_j - \beta_g) \right|.$$

The truthful messaging condition only characterizes one aspect of a group's strategy: whether it is able to credibly communicate with association R. I now move on to a condition that I term *voluntary association.* This condition characterizes the requirement that each group in association R would prefer remaining in R to exit. Let $V(\theta|m)$ be the expected variance of a posterior belief about θ after receiving m signals. The assumption of a quadratic utility function, combined with sequential rationality, enables us to express the equilibrium expected utility to player g from action profile $\sigma = \{E, D, X\}$ as the following:

$$EU_g(y_\sigma) = -V(\theta|n_g)$$

for $g \in X$, and[12]

$$EU_g(y_\sigma) = -\sum_{k \in R} \alpha_k^R (\beta_k - \beta_g)^2 - \sum_{h \in E} \alpha_h^R V(\theta|n_E) - \sum_{j \in D} \alpha_j^R V(\theta|n_E + n_j) \tag{6}$$

for $g \in R = E \cup D$.

The voluntary association condition requires that for each $g \in R$, the expected utility from remaining in R exceeds the expected utility g would receive from forgoing the association in favor of implementing its own policy. The condition also requires that each $h \in X$ faces no possible

---

[12]Recall that a group choosing exit can perfectly target its choice of policy y_g to its bias, β_g, and so the group's expected utility is only a function of the variance surrounding its own decision.

benefit from joining the association. This requires that for each group that has chosen exit from some association R, there is no messaging equilibrium for association $R' = R \cup \{h\}$ that yields h strictly higher expected utility than would be received by remaining out of the association. For the statement below, let $R' = R \cup \{h\}$. We can now express the voluntary association condition as follows.

Condition 2 *The **voluntary association condition for group** $\mathbf{g} \in \mathbf{R}$ requires that:*

$$-\sum_{k \in R} \alpha_k^R (\beta_k - \beta_g)^2 - \sum_{h \in E} \alpha_h^R V(\theta | n_E) - \sum_{j \in D} \alpha_j^R V(\theta | n_E + n_j) > -V(\theta | n_g).$$

*The **voluntary association condition for group** $\mathbf{h} \in \mathbf{X}$ requires that:*

$$-V(\theta | n_h) \geq \max_{\{E', D'\} \in \mu(R')} -\sum_{k \in R'} \alpha_k^{R'} (\beta_k - \beta_h)^2 - \sum_{h \in E'} \alpha_h^{R'} V(\theta | n_{E'}) - \sum_{j \in D'} \alpha_j^{R'} V(\theta | n_{E'} + n_j).$$

One immediate implication of Condition 2 is that, if $\sum_g \alpha_g^R \geq 1$ for all associations R, there can never be a stable configuration of groups in which the set of engagers, E, is empty but the association is nonempty. If this occurs, then the group with the most information can always benefit from exit; if it exits, this group can both make a more informed decision than the other groups and perfectly target policy to its own bias. This is stated in the following corollary.

Corollary 1 *Let $\sum_g \alpha_g^R \geq 1$ for all associations R. If $\sigma = \{E, D, X\}$ is such that $E = \emptyset$ and $D \neq \emptyset$, then σ cannot be stable and, in particular, will violate voluntary association for some group in $g \in D$.*

3.1 Social welfare

For a given collection of groups, Conditions 1 and 2 can be used to completely characterize the set of stable societal configurations of groups into sets E, D and X. There always exists at least one stable configuration of groups and this is $\sigma = \{\emptyset, \emptyset, G\}$, in which every group has chosen

exit. In this case the association is empty, and so a group choosing "association" will simply be associating with itself. As the group is indifferent between entry and exit, by Definition 2 exit is a stable choice. Oftentimes, however, there are multiple stable configurations of groups. Galeotti et al. demonstrate that when multiple equilibria exist in their framework they can be Pareto-ranked and characterized by a straightforward rule. The spirit of their result translates in part to the setting considered here. For a fixed association R the set of messaging equilibria $\mu(R)$ can similarly be Pareto-ranked, although the rule that characterizes the ranking in Galeotti et al. no longer holds here because of the varying weights α attached to the policy decisions of the groups. However, when multiple equilibria exist that correspond to *different* stable associations, these equilibria can oftentimes not be Pareto ranked. This stems from the nature of the association decision, and the fact that any group choosing exit obtains the minimum level of utility they could be expected to receive. Therefore, if R and R' are different associations that correspond to stable equilibria σ and σ' respectively, and if $g \in R \setminus R'$ and $j \in R' \setminus R$, then σ and σ' cannot be Pareto ranked: g prefers σ to σ' and j prefers σ' to σ.

As shown earlier, g's ex ante expected utility to association with R is

$$EU_g(y_\sigma) = -\sum_{k \in R} \alpha_k^R (\beta_k - \beta_g)^2 - \sum_{h \in E} \alpha_h^R V(\theta|n_E) - \sum_{j \in D} \alpha_j^R V(\theta|n_E + n_j).$$

The first term in this equation is constant for a given g and R while the next two terms are dependent on the messaging strategies of the members of R and are the same for all members of R. Thus, messaging equilibria $\mu(R) = \{\{E_1, D_1\}, \{E_2, D_2\}, ...\}$ are Pareto ranked on the basis of maximization of

$$-\sum_{h \in E} \alpha_h^R V(\theta|n_E) - \sum_{j \in D} \alpha_j^R V(\theta|n_E + n_j).$$

The term $V(\theta|m)$ reduces to the following fraction, leading to the following result:

$$\int_{\theta=0}^{1} \sum_{k=0}^{m} \binom{m}{k} \theta^k (1-\theta)^{(m-k)} \left(\frac{(k+1)(m-k+1)}{(m+2)^2(m+3)} \right) = \frac{1}{6(m+2)}.$$

Proposition 1 *For a fixed association R, messaging equilibrium $\{E, D\}$ Pareto dominates $\{E', D'\}$*

20

if and only if

$$-\sum_{h \in E} \frac{\alpha_h^R}{6(n_E + 2)} - \sum_{j \in D} \frac{\alpha_j^R}{6(n_E + n_j + 2)} > -\sum_{h \in E'} \frac{\alpha_h^R}{6(n_{E'} + 2)} - \sum_{j \in D'} \frac{\alpha_j^R}{6(n_{E'} + n_j + 2)}.$$

The following corollary follows directly:

Corollary 2 *Suppose* $\sigma = \{E, D, X\}$ *and* $\sigma' = \{E', D', X'\}$ *are stable configurations with* $X = X'$. *Then:*

1. If $E' \subset E$ then σ Pareto dominates σ'.

2. If $n_g = n_j$ and $\alpha_g = \alpha_j$ for all g, j then σ Pareto dominates σ' if and only if $n_E > n_{E'}$.

3. If $n_g = n_j$ for all g, j and if $n_E > n_{E'}$ then σ Pareto dominates σ'.

4. If $n_g = n_j$ for all g, j and $n_E = n_{E'}$ then σ Pareto dominates σ' if and only if
   $$\sum_{g \in E} \alpha_g < \sum_{j \in E'} \alpha_j.$$

While stable equilibria corresponding to different associations can generally not be Pareto ranked, it is always the case that members of an association receive a higher level of expected utility by associating than by exit. If institutional factors such as policy weights α can be utilized to increase the size of a stable association, then this new equilibrium will dominate the former equilibrium with respect to something similar to Rawlsian (max-min) social welfare, in that the new association raises the expected utility of one or more groups above minimum levels. Section 5 works through an example in which institutional mechanisms can be varied in an attempt to grow an association.

3.2 Two groups

Before concluding this section, it is useful to consider the (much simplified) conditions for truthful messaging and voluntary association for the case of two groups, g and h. In this setting, truthful messaging is satisfied for group g if

$$\frac{1}{2(n_g + n_h + 2)} \geq |\beta_h - \beta_g|.$$

Clearly this condition holds for group g if it also holds for group h; thus, regardless of preference divergence, regardless of the relative sizes of the two groups and regardless of the relative policy-making authority of the two groups (α), each group faces the same incentive to message truthfully and the incentive is wholly dependent on the preference divergence of the two groups and the total size of the population, $n_g + n_h$. Of course, there may be stable configurations of two groups in which one engages and one disengages; in these cases, however, since the truthful messaging condition is satisfied for both groups and both have chosen to associate, the configuration is Pareto dominated by another in which both choose to engage.

Group-level differences arise, however, when evaluating incentives to associate or exit, and the α terms come into play in this evaluation. If both groups message truthfully the voluntary association condition for group g is

$$\alpha_h(\beta_h - \beta_g)^2 \leq \frac{1}{6(n_g + 2)} - \frac{1}{6(n_g + n_h + 2)},$$

which requires that the weighted squared distance of their biases be less than the expected variance of θ conditional on n_g signals minus the expected variance of θ conditional on $n_g + n_h$ signals. If both groups babble then the voluntary association condition cannot be met for both groups, by Corollary 1.

With two groups the truthful messaging condition will, in general, bind before the voluntary association condition. In other words, when both groups have an incentive to message truthfully to each other within an association, they virtually always prefer association (for a messaging equilibrium where both message truthfully) to exit. In particular, they always prefer association to exit when each group contains two or more individuals, or when the α terms are proportional to group population, so that $\alpha_g = \frac{n_g}{n_g + n_h}$ and $\alpha_h = \frac{n_h}{n_g + n_h}$. At the same time, if relative policymaking discretion, α, is highly disproportional to the groups' relative sizes and one group contains a single individual, then examples can be constructed in which mutual engagement is a messaging equilibrium but is not stable; the larger group prefers exit to association.

4 Association dynamics

The truthful messaging condition defined in Condition 1 has a clear interpretation: if the impact of a sender's lie on the activities of others is greater than (a function of) the preference divergence between the sender and the other members of its association, then that lie shifts the activities of the other groups too much. In this case, lying is not beneficial, and truthful communication from the sending group to the association can be sustained in a messaging equilibrium. In this section I consider the special case of three groups in order to provide some basic intuition for the equilibrium dynamics of truthful messaging, association and exit. The three group setting provides the simplest illustration of the effect of one group on the dynamics of a preexisting association (or non-association). More specifically, the setting enables us to consider some simple comparative statics: pinning down the size, policymaking discretion and locations of two groups we can study how the presence of the third group alters association and communication strategies as we vary parameters that characterize the third group.

The following examples illustrate two cases of interest; in the first, the entry of a third group breaks a preexisting association between groups 1 and 2. In the second, the third group's entry enables association between these groups where previously association was impossible. In both, the shaded regions of the corresponding figures depict the groups that have chosen association.

Example 1 *A "moderately extreme" third group may hinder beneficial association.*

Suppose that groups 1 and 2 are identical with respect to size and that policymaking discretion (α) is directly proportional to the size of each group. The example uses parameter values $\beta_1 = 0; \beta_2 = .05; n_1 = n_2 = 2$ and $n_3 = 1$. At these parameter values 1 and 2 have biases that are sufficiently close to each other to enable truthful messaging within association $R = \{1, 2\}$.

Starting at $\beta_3 = 0$, Figure 1 shows a Pareto-optimal and stable equilibrium for each value of $\beta_3 \geq 0.$[13]

---

[13]For some values of β_3 there are multiple Pareto-optimal equilibria, but all Pareto-optimal equilibria yield identical ex-ante expected utility to the groups.
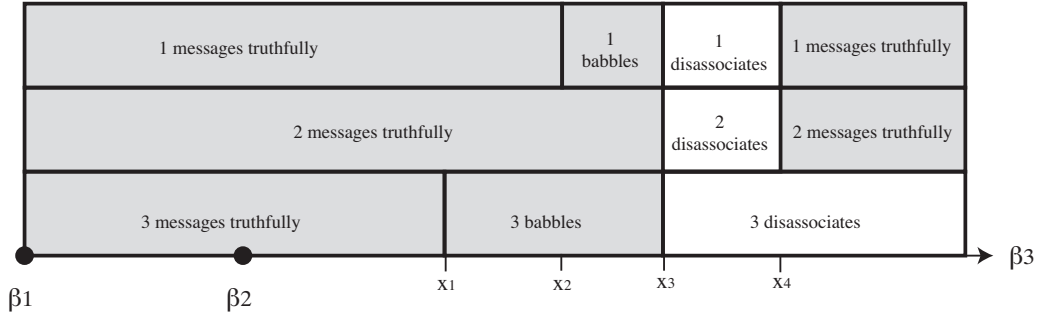
Figure 1: "Spoiler" equilibria as group 3 becomes sufficiently extreme.

- Beginning at $\beta_3 = \beta_1 = 0$, truthful messaging can be supported for all three groups until the point at which β_3 reaches x_1.

- At the point $\beta_3 = x_1 = .096$, group 3 becomes too distant from groups 1 and 2 to be able to communicate truthfully to them. Group 3 still wishes to listen to the other groups; the messages from 1 and 2 both reduce variance in its own choice and in those groups' choices. However, when group 3 babbles the value of association for all groups is reduced, as less information is being communicated.

- At $\beta_3 = x_2 = .149$ group 3 becomes too distant for group 1 to be able to communicate truthfully to an association containing group 3. At this point, both 1 and 3 remain in the association in order to listen to group 2, which is still incentivized to communicate truthfully to 1 and 3. At the same time, with fewer signals being communicated to the association, the value of association again decreases.

- At $\beta_3 = x_3 = .176$ the value of association for group 1 becomes negative, and group 1 exits the association. While truthful communication by 2 to groups 1 and 3 is a messaging equilibrium for $R = \{1, 2, 3\}$ it is not stable, as 1 wishes to exit (although 3 wishes to remain in the association). At the same time, while truthful communication between groups 1 and 2 is a messaging equilibrium for $R = \{1, 2\}$ it is not stable, as group 3 would want to join that association. When group 1 exits the association, truthful communication cannot be sustained

24

between groups 2 and 3. Therefore there is only one stable equilibrium, and it corresponds to exit by all groups.

- Finally, at $\beta_3 = x_4 = .186$ group 3 would no longer wish to be in an association with groups 1 and 2. From this point on groups 1 and 2 can form a stable association without a threat of entry by 3.

Example 2 *A third group may induce beneficial association.*

In the previous example groups 1 and 2 were kept from communicating with each other by the presence of a third group. In the following example the third group is able to induce association between 1 and 2 where previously those groups could not associate. As before, suppose that groups 1 and 2 are identical with respect to size ($n_1 = n_2 = 2$), that $n_3 = 1$ and that the α terms are proportional to group size. However, now let $\beta_1 = 0$ and $\beta_2 = .085$; these biases are too far apart for truthful messaging to be sustained within association $R = \{1, 2\}$. Again, for each $\beta_3 \geq 0$, Figure 1 shows a Pareto-optimal and stable equilibrium.[14]

- Beginning at $\beta_3 = \beta_1 = 0$, truthful messaging can be supported by groups 1 and 3; group 2 is too distant to communicate truthfully, but associates with 1 and 3 in order to utilize their information.

- From $\beta_3 = x_1 = .040$ until $\beta_3 = x_2 = .044$, group 3 is sufficiently moderate with respect to β_1 and β_2 that truthful messaging can be sustained between all three groups.

- As β_3 moves above $x_2 = .044$ group 3 becomes too distant for group 1 to be able to communicate truthfully to an association containing groups 2 and 3. 1 remains in the association in order to listen to 2 and 3, which are sufficiently close to each other to be incentivized to communicate truthfully to association $\{1, 2, 3\}$.

---

[14]Again, for some values of β_3 there are multiple Pareto-optimal equilibria, but all Pareto-optimal equilibria yield identical ex-ante expected utility to the groups.
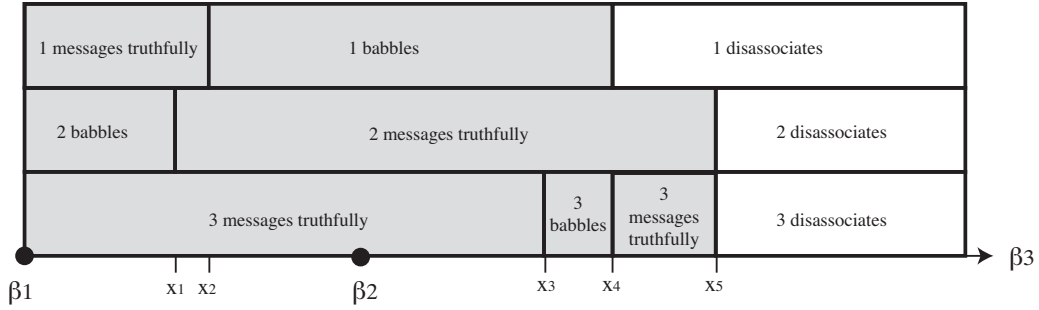
Figure 2: Group 3 induces (beneficial) universal association.

- At $\beta_3 = x_3 = .13$ group 3 has become too distant from 1 and 2 to be able to communicate truthfully to them and resorts to babbling. However 2 still communicates truthfully to 1 and 3 and the value of association remains positive for all groups.

- At $\beta_4 = .147$ the value of association for group 1 becomes negative, and group 1 exits the association. With 1's exit group 3 is now able to communicate truthfully to 2 (and vice-versa, as in the 2-group case the truthful messaging condition is identical for both groups).

- Finally, at $\beta_3 = x_5 = .185$ truthful communication can no longer be sustained between groups 2 and 3. At this point both exit and the unique stable equilibrium corresponds to exit by all groups.

5 Using influence as an inducement

The α terms can be used to incentivize certain groups to both associate and to truthfully communicate. In the example that follows, let there be three groups with $n_1 = 2, n_2 = 4$ and $n_3 = 1$, and with $\beta_1 = 0$, $\beta_2 = .07$ and $\beta_3 = .17$. In this case group 2 is both considerably larger than groups 1 and 3, and the smallest group, group 3, is a preference outlier in the sense of being farther from median group 2 than group 1 is. In this example any association containing any pair of groups cannot sustain truthful communication; each group is simply too far from any other group and/or

26

too well-informed. At the same time, groups 1 and 3 would benefit from association with group 2 to the extent that yielding all policy-making authority to 2 would be preferable to exit for these groups.

While group 2's truthful communication and voluntary association conditions trivially hold at $\alpha_2 = 1$ and $\alpha_1 = \alpha_3 = 0$, the equality is knife-edged; letting $\alpha_1 = .001$, $\alpha_2 = .998$ and $\alpha_3 = .001$, for example, breaks 2's incentive to associate. Moreover, there may be normative reasons to grant a positive degree of policy-making discretion to every group, in keeping with Young's argument that "...a democratic decision is normatively legitimate only if all those affected by it are included in the process of discussion and decision-making."[15] The question then is whether there exists a strictly positive vector of policy-making weights, α, for which all groups have a strictly positive incentive to associate, and for which some information can be shared across groups. In this example I assume $\sum_i \alpha_i = 1$, so that there is no artificial benefit ($\sum_i \alpha_i < 1$) or penalty ($\sum_i \alpha_i > 1$) stemming from the association decision.

To answer this question, first note that there is no strictly positive α than can ever induce either groups 1 or 3 to truthfully communicate to an association consisting of $\{1, 2, 3\}$ (the verification of this statement, and all further calculations for this example, are in the appendix). It follows that the only possible association consisting of all three groups would correspond to $\{2\} = E$ and $\{1, 3\} = D$. If voluntary association is nontrivially satisfied for 2 at some α then, by Corollary 1, it must be the case that 2 is incentivized to communicate truthfully to 1 and 3. If this is the case, then 2's association condition requires that $\alpha_1 \geq 2.95\alpha_3$. However, satisfaction of the association requirement does not imply that $\{\{2\}, \{1, 3\}\}$ is a messaging equilibrium for association $\{1, 2, 3\}$. To see this, note that when $\alpha_3 = 0$ the above equation is satisfied for 2 but, as 2's message only has a tangible effect on 1's action, truthful communication cannot be sustained. In this case 2's truthful messaging condition is identical to its messaging condition within an association consisting of only 1 and 2 and, by design, the condition is not satisfied for the pair. Group 2 would want to associate with groups 1 and 3 if 2 could credibly communicate to them, but 2's message is not

---
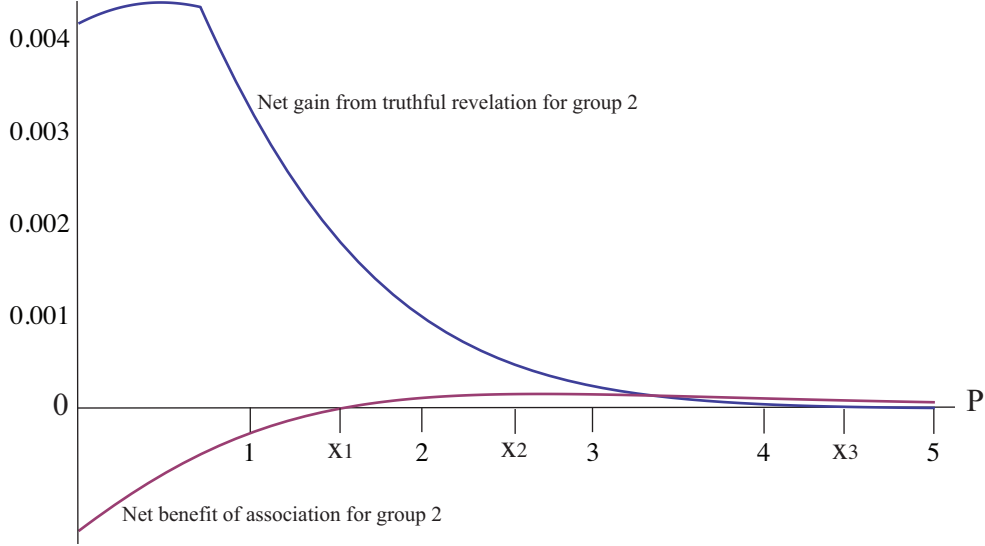
[15] Young (2002, p. 23).

27

Figure 3: Varying an index of proportionality changes 2's incentives.

credible. Satisfaction of truthful communication for 2 additionally requires that the following pair of inequalities holds: $.25\alpha_3 \leq \alpha_1 \leq 26.12\alpha_3$.

To summarize, there *does* exist a collection of strictly positive policy-making weights α for which the association $E = \{2\}, D = \{1,3\}$ is stable, and for which each group's benefit to association is strictly positive. Letting Δ_+^2 be the interior of the 2-dimensional unit simplex, this set is defined as $\{(\alpha_1, \alpha_2, \alpha_3) \in \Delta_+^2 : 2.95\alpha_3 \leq \alpha_1 \leq 26.12\alpha_3\}$.

To see how varying the distribution of α across the three groups affects group 2's association and messaging conditions I present two figures. In Figure 3 I explicitly plot out the net benefits of truthful messaging and association for group 2 as a "proportionality index" P is varied. For a given association R, let p_i be the proportion of the population in group i, or $p_i = \frac{n_i}{\sum_{j \in R} n_j}$. P then generates α as follows: $\alpha_i(P) = \frac{p_i^P}{\sum_{j \in R} p_j^P}$. Thus, at $P = 0$ policy-making authority is equally distributed across the groups, irrespective of size; in this example $P = 0$ corresponds to $\alpha(0) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. At $P = 1$ authority is directly proportional to group size, so that $\alpha(1) = (\frac{2}{7}, \frac{4}{7}, \frac{1}{7})$. As $P \to \infty$, α approaches $(0, 1, 0)$.

Figure 3 shows that for low values of P truthful messaging can be sustained for 2, but voluntary

association cannot; the reason for this is that when 3 is given too much authority relative to 1, 2's utility decreases in two ways; first, 2 suffers greater disutility from 3's more extreme bias relative to 1, but second, 2 also loses the benefit of 1's extra signal relative to 3. Since 1 and 3 cannot message truthfully, the value of their information is only realized through their own activity choices. Thus, 2 benefits doubly when 1 is granted greater authority than 3. In Figure 3 the bottom curve represents 2's net gain from association versus exit while the top curve represents 2's net gain from truthful communication. The figure shows that 2 is incentivized to associate only when P exceeds $X_1 = 1.56$, which corresponds to $\alpha(1.56) = (.23, .69, .08)$. 2 receives its maximum benefit from association when P reaches $X_2 = 2.7$, corresponding to $\alpha(2.7) = (.13, .85, .02)$. As noted earlier, when 2 is granted all policy-making authority it is indifferent between association and exit. That the value of association for 2 is maximized at an interior point reflects the fact that 2 can strictly benefit from the decisions made by 1 and 3 when those groups are given the additional information that 2 possesses.

When P reaches $x_3 = 4.7$, group 2's truthful messaging condition fails to hold; at this point group 1 is receiving too much authority relative to 3 for 2 to be incentivized to be truthful. Prior to this point, 2's incentive to manipulate 1's choice was kept in check by the fact that such manipulation would also generate a costly change in 3's choice. As 3's authority becomes too low relative to 1's, 3's presence can no longer prevent 2 from benefiting from manipulating 1. This point approximately corresponds to $\alpha(4.7) = (.037, .962, .001)$.

Figure 4 depicts the Pareto-optimal, stable equilibria for all possible realizations of α. The vertices of the simplex represent the values of α at which one player is granted all policy-making authority, with the top of the simplex representing the point at which 2 has all authority. The collection of α's characterized by proportionality index $P \geq 0$ is represented by the curve. Although difficult to see in the figure, this curve leaves region II as it approaches $\alpha = (0, 1, 0)$. The shaded region II represents the values of α for which 2 truthfully messages to 1 and 3, and for which the association decision is positive for all groups. In region I group 2 cannot be compelled to truthfully communicate to 1 and 3, because (as described above) α_3 is too low relative to α_1 for group 3 to
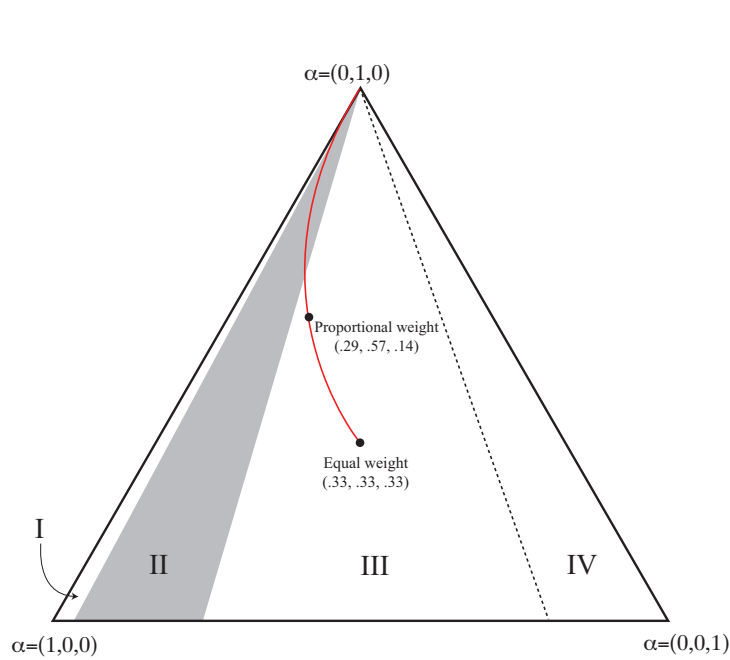
Figure 4: 2's incentives for all possible distributions of α.

effectively counterbalance group 1's bias. In region III group 2 *can* truthfully communicate with 1 and 3, but prefers exit to association. Again, as described above, in this region α_3 is too large relative to α_1 for association to be beneficial to 2, because a large α_3 requires 2 to endure 3's greater bias and lower information. In region IV, group 2 can no longer credibly communicate with 1 and 3, because α_1 is too low relative to α_3 for group 1 to counterbalance group 3's bias.

Interestingly, the absolute levels of α_1 and α_3 don't affect 2's decision to either associate or communicate truthfully; only the relative levels of these terms matter. 2 can be incentivized to communicate truthfully and strictly prefer association to exit even when it is granted zero policy-making authority. It is also important to note that truthful messaging by 2 and voluntary association by all groups can only be meaningfully sustained when both $\alpha_1 > 0$ and $\alpha_3 > 0$. Significantly, even though group 3 is a preference outlier and possesses little information of its own, its nontrivial presence in the association is necessary in order to induce 2 to communicate truthfully.[16]

---

[16]This example is qualitatively similar to results found in Iaryczower and Oliveros (2013). Those authors show that in a model of decentralized bargaining with vote buying and selling, beneficial "power brokers" can emerge. These

6 Foregoing or obfuscating information

If the truthful messaging condition fails for a group it is not difficult to find cases in which a group could profitably forego some of its own information in order to make its messages more credible to an outgroup (and in which giving up this information is a Pareto improvement). Consider a two group example in which $n_1 = n_2 = 7$, $\beta_1 = 0, \beta_2 = .033$ and $\alpha_1 = \alpha_2 = .5$. Truthful communication between groups requires $\beta_2 \leq .03125$ and so the only stable configuration of groups is one in which both 1 and 2 exit and receive expected payoffs of $-.0185$. However, if 2 commits to giving up a signal–essentially ignoring some of its own information–messaging can be sustained for $\beta_2 \leq 0.03333$. The payoff to association in this case is $-.01165$, and so 2 is made strictly better off: by giving up one of its own signals 2 is able to gain the 7 signals of group 1.

At the same time, this comparison is unsatisfying because there may exist other messaging equilibria that Pareto dominate the equilibrium 2 generates by foregoing a signal. Up until now I have focused exclusively on fully separating equilibria in which a group sending a message to an association must be incentivized to be truthful for any number of positive signals it has received; if the group wishes to misrepresent its true number of positive signals, all other groups receiving its message assume the message is uninformative. The focus on separating equilibria clearly simplifies the analysis, and, importantly, the substance of the results presented so far would not change if I expanded my scope to consider semi-separating equilibria. The dynamics in the examples I have presented stem from the tension between the "congestion effect" of more signals (more signals make the effect of a lie smaller, and often reduce incentives for truthful messaging),[17] and from the

---

brokers serve as middlemen to facilitate transactions between parties that would not otherwise negotiate with each other. A necessary condition for this scenario to emerge is that the broker has a stake in the policy outcome. If we conceive of group 2 as such a middleman, this example bears some resemblance to their result, in that 2's presence induces 1 and 3 to join the association, and that this presence can be manipulated through what could potentially be conceived of as vote trading. The example also bears a connection to a "mediator"-type result in Patty and Penn (2013)'s three-player game of networked, sequential decision-making.

[17]As noted by Galeotti et al. (2013), and in contrast to other work finding a similar congestion effect (e.g. Morgan and Stocken (2008)), incentives for truthful communication do not necessarily decrease in the amount of information

"association effect" of more signals (more signals reduce variance, and thus increase incentives for association). Both effects will also be present if considering semi-separating equilibria.

In this example, the equilibrium resulting from 2 foregoing a signal does not outperform the best semi-separating equilibrium. In the Appendix I derive the best partitional equilibrium for the parameters given above, and find that it marginally outperforms the separating equilibrium in which group 2 gives up a signal. While formally defined in the Appendix, this partitional equilibrium can be informally described for each group as follows: if the group receives 0 or 1 positive signals it sends message m^1; if it receives 2 positive signals it sends m^2; if it receives 3 or 4 positives it sends m^3; if it receives 5 positive signals it sends m^4; and if it receives 6 or 7 positive signals it sends m^5. Thus, each group introduces some noise into its signal; this noise prevents a lie from being too profitable to the sending group while also conveying beneficial information to the receiving group.

In this example truthful messaging can only be sustained in equilibrium when $\beta_2 \leq .03125$. The partitional equilibrium described above can be sustained for $\beta_2 \leq .0408$, and when $\beta_2 = .033$ it can be shown to represent the best partitional equilibrium for both groups. The expected utility received by each group at this equilibrium equals $-.011634$.

Returning to our original question of whether the semi-separating strategy represents an improvement for 2 relative to foregoing a signal: in this case it does. The groups received an expected utility of $-.01165$ when 2 gave up a signal, but receive $-.011634$ when playing the best partitional strategy. Whether welfare could be improved in this game by a reduction in information remains an open question when considering the possibility of semi-separating equilibria; in this example the net payoff to the partitional equilibrium relative to the equilibrium corresponding to a lost signal is small relative to the payoffs generated by other equilibria. If the answer is "yes" an example will likely require three or more groups, and stem from the fact that while a partially informative messaging strategy can always be used to conceal information from a receiving group, that strategy cannot prevent the sending group from observing all of its own signals, and thus cannot mitigate the congestion effect stemming from this private information.

---

held by a receiver.

7 Limiting within-group communication

The previous section explored one possible intra-group mechanism for improving inter-group communication: in the event that truthful communication cannot be sustained across groups, it can potentially be beneficial for a group to "give up a signal" in order to reduce the congestion effect of too much information and to enable communication. At the same time, that section showed that, in the example provided, foregoing a signal in this way did not represent an improvement over the group partially revealing its information through a semi-separating strategy. In this section I consider a different intra-group mechanism for improving communication across groups: limiting intra-group communication prior to the public messaging stage. Consequently I set aside the conception of the group as a unitary actor for the remainder of this section, and this will be shown to be in the group's interest.

Suppose there are two groups, 1 and 2, with $\beta_1 = 0$ and $\beta_2 > 0$. Truthful messaging for these groups requires that

$$\frac{1}{2(n_1 + n_2 + 2)} \geq \beta_2.$$

Now suppose that each group prohibits intra-group communication prior to the public messaging stage. In this case, each person approaches the association with only their own signal to reveal. Last, suppose that policy-making authority has been disaggregated within each group, so that the activity choice of each $i \in g$ receives weight α_{gi}, with $\sum_i \alpha_{gi} = \alpha_g$ and with $\alpha_1 + \alpha_2 = 1$.

The condition for truthful messaging by an individual $i \in g$ to the association consisting of all members of both groups is now

$$\sum_{j \in h} \frac{\alpha_{hj}}{2(n_1 + n_2 + 2)^2} + \sum_{j \in g \setminus \{i\}} \frac{\alpha_{gj}}{2(n_1 + n_2 + 2)^2} \geq \sum_{j \in h} \frac{\alpha_{hj}}{(n_1 + n_2 + 2)} \beta_2$$

which reduces to

$$\frac{1}{2(n_1 + n_2 + 2)^2} - \frac{\alpha_{gi}}{2(n_1 + n_2 + 2)^2} - \frac{\alpha_h}{(n_1 + n_2 + 2)} \beta_2 \geq 0. \tag{7}$$

Equation 7 shows that, with two groups, when information is disaggregated within each group, truthful messaging is easier to achieve; group members are more incentivized to be truthful because

33

a false message not only biases the choices of members of the outgroup, but also biases members of their own group in an undesirable way.[18] This is akin to making the group's signal costly, and bears some similarity to the models developed in Patty and Penn (2013) and Gailmard and Patty (2013). Equation 7 also demonstrates that there is an optimum distribution of authority, both across groups and within groups, that maximizes the incentive for universal communication. Since the messaging condition binds more strongly as α_{gi} and α_h increase, the condition is easiest to satisfy when the maximum value these terms could take is minimized, which occurs at $\alpha_1 = \alpha_2 = \frac{1}{2}$, and $\alpha_{gi} = \alpha_{gj} = \frac{\alpha_g}{n_g}$. Thus, an even distribution of power across individuals maximizes incentives for universal communication in the two-group case.

To show that limiting intra-group communication in this way is particularly beneficial, I return to the example of the best semi-separating equilibrium presented in Section 6. In the example, $n_g = n_h = 7$, $\beta_2 = .033$ and $\alpha_1 = \alpha_2 = \frac{1}{2}$. The example showed that in the optimal semi-separating messaging equilibrium signals are mapped into five messages. The expected utility to each player from association is $-.011649$. In the example truthful messaging (a separating messaging equilibrium) cannot be sustained for $\beta > 0.03125$. However, if intra-group communication is prohibited prior to the public message, Equation 7 reveals that truthful messaging for all individuals can be sustained for $\beta < .058$ (when $\alpha_i = \frac{1}{14}$ for each i). Moreover, since all information is transmitted to all groups, the equilibrium with disaggregated information is payoff equivalent to truthful messaging between groups 1 and 2 (which could not be sustained in a separating equilibrium with aggregated information). This equilibrium necessarily Pareto dominates the best semi-separating equilibrium, as it involves more information transmission. In particular, it yields each member of the association an expected utility of $-.01096$.

---

[18]Disaggregating information in this way may no longer be *unambiguously* beneficial when there are three or more groups, for reasons related to the congestion effect described in Footnote 17. However, it will certainly be beneficial in many instances.

8   Discussion

In a section of his book entitled "Demes and Tribes as Social Networks," Joseph Ober describes a radical process of institutional change undertaken by Cleisthenes, the newly instated leader of Athens, following the Athenian Revolution of 508 B.C.[19] At this time Athenian society was composed of numerous disconnected *demes* (towns, villages, or urban neighborhoods) in which residents had strong ties to each other but limited contact with outside groups. Cleisthenes realized that in order to grow the capacity of the state, and in particular, military readiness for a possible Spartan attack, some mechanism was needed to induce individuals to develop "bridging ties" of communication across these close-knit communities. His solution was the formation of a new political system founded upon ten new, artificially created (and territorially non-contiguous) tribes, each named after a mythical hero. Each deme of Athens was assigned to one of these ten new tribes, and each tribe was engineered to draw a third of its membership from coastal, inland and urban territories in Athens. By design, the new tribes were deliberately distinct from the four kinship-based Ionian tribes to which virtually all Athenians belonged.

In addition to this new tribe system, Ober describes the "Council of 500," an agenda-setting body that was created at the same time to perform the essential task of deciding upon the issues that would be considered by the full Assembly of Athenian citizens. The council was composed of ten fifty-member delegations, one from each new tribe, with the composition of each delegation proportional to the population of each deme within the tribe. Delegates served non-renewable one-year terms, and each delegation took up temporary residence in the city for the year during which it was selected to serve. Last, each tribal delegation was responsible for leading the Council for one-tenth of the year, with the Council leadership rotating through all ten tribes over the course of the year. At the end of the year, delegates returned home to their demes.

How might this new political organization have fostered communication and learning in ancient Athens? Ober argues that several features made this system particularly successful at getting people to communicate with each other. In a rich account of what a year in the life of a hypothetical

---

[19]Ober, pp. 134-51.

delegate might have looked like, Ober focuses on opportunities that existed for enterprising individuals to serve as bridges between groups that might not have previously communicated with each other, and the social capital that would accrue to those individuals and others who emulated them. The existence of term limits meant that these bridging opportunities were recreated year after year, as each new batch of delegates developed its own network of communication and expertise.

The results in this paper hint at different processes that might have additionally fostered communication in Athens. By drawing their memberships equally from coastal, inland and urban demes, the artificial tribes created by Cleisthenes ensured that individuals were grouped with other like-minded individuals *with whom they had no previous ties of communication*. In other words, assuming that demes from coastal regions might have held similar biases but had little day-to-day contact with each other, the presence of multiple such groups within a tribe may have made the misrepresentation of information costlier. The intuition for this was presented in Section 7's example of the benefit of disaggregating information within groups; when like-minded individuals enter an association without having already shared information, then a false message not only biases those with opposing preferences but also those with similar preferences. This can be costly for the group tempted to lie.

Both examples presented in Section 4 show that with three types of groups there are opportunities for "bridging groups" to foster communication and association across communities that would not have otherwise talked or associated with each other. Sometimes the bridging group can foster truthful communication between all three groups while in other cases it may not be possible for all groups to simultaneously be truthful. At the same time, the amount of beneficial communication that does occur is sufficient to make association preferable to exit for everyone. Thus, the entrepreneurial "bridging delegates" described by Ober might have succeeded in fostering truthful communication across all groups, or they might have simply succeeded in providing enough credible information themselves so as to induce others to remain in the association as listeners.

Modern examples of similar organizational issues can be found in Albert Hirschman's book *Exit, Voice and Loyalty*, which argues that two important forces in stemming an organization's

decline are "exit" and "voice." Individuals choosing "exit" withdraw from a declining organization, whereas those choosing "voice" attempt to mitigate the decline by expressing dissatisfaction with the organization's performance, and by possibly suggesting remedies. A major theme of the book is that the cost of exit (borne by an individual or group) has important and complicated implications for the success of an organization. In many examples, this cost can be thought of as "loyalty."

Some of these themes, and indeed, some specific examples presented by Hirschman, can be thought of in the context of the model presented here. In the case of public education, for example, it could be that high costs to exit generate worse outcomes than either costless exit or simply disallowing exit altogether. This is because costly exit (which could be represented, for example, by a situation in which the α_i terms sum to less than one) may result in a system in which only the most informed groups can benefit from enrolling their children in private schools. This type of scenario could be thought of in the context of the example in Section 5, in which the presence of group 2 in an association would be a tremendous benefit to the other groups, but the quality of group 2's information tempts it to withdraw from the association.

On the other hand, Hirschman notes, too much loyalty can also hinder organizational performance. Hirschman argues that this is because organizations that seek to promote loyalty may also seek to stifle voice, as both strategies promote organizational slackness. Conversely, exit and voice serve as complements in that organizations may promote voice (and the beneficial information it provides) as exit becomes easier. This model presents some additional reasoning for Hirschman's argument. If exit is too costly, then groups that would otherwise not want to associate with each other could be artificially induced to do so. This could result in a scenario in which association is beneficial not because of the incentives for information transmission between members (or "voice") but rather, because of loyalty, or even coercion. In particular, these artificially large associations might be less capable of sustaining truthful communication than smaller associations would be. However, as in Hirschman's book, the implications of the model are complex; examples can be constructed in which loyalty is good or bad, depending on the incentives of the members. Loyalty that keeps a high-information member within the organization may be productive, whereas

loyalty that keeps a high bias member in the organization may stifle voice.

9 Conclusions

In this paper I considered the possibility of information transmission across groups in a setting in which groups hold different biases and different amounts of information, any group may choose to unilaterally exit society, and communication is costless. The paper characterizes the "associations of groups" that can be sustained in equilibrium and, within those associations, the types of groups that choose to engage in or disengage from the process of inter-group communication. These associations are the collections of groups for which some information transmission between groups is possible, and for which the benefits of this information outweigh the costs stemming from preference diversity within the association.

Several insights emerge from the model. First, the presence of a third group may induce association between two groups that could not associate previously, even if the third group's bias is more extreme than the biases of the other two. At the same time, situations can also arise in which the presence of a third group can extinguish any possibility of communication between groups, where previously communication was possible. Second, institutional mechanisms governing the amount of policy discretion held by each group can be used as a lever to induce beneficial communication and universal association, even in settings in which association is impossible between any pair of groups. These mechanisms can be highly disproportional, and may need to be calibrated to ensure satisfaction of both the association and messaging conditions for the group(s) the institutional designer wishes to induce to "talk." And third, certain mechanisms may be used *within* a group in order to make that group's message more credible to an association. These mechanisms include excluding some information from a group's own pool of information (by, for example, refusing to acknowledge a group member's signal); obfuscating information; and limiting within-group communication, so that information transmitted to an outgroup must also be transmitted to the ingroup (thus rendering a lie more costly).

38

As discussed in the previous section, the model can be used to help us better think about organizational design in situations in which an organization's membership is fluid. While exogenous costs and benefits to association are not explicitly analyzed in this paper, it is generally regarded that these factors (e.g. "exit" and "loyalty") are important determinants of organizational performance. Developing these ideas, along with richer notions of a group's association and communication possibilities, are left for future research.

10 Appendix

Proof of Lemma 1.

If group g has n_g members then it can state that its members have received any number of signals in $\{0, ..., n_g\}$. We start by supposing that revealing some $\tilde{s} > s_g + 1$ represents a profitable lie for g to make, in the sense of biasing the information of the other groups in a way that is attractive to g. We will show that if this is the case, then it must be the case that revealing $s^o = s_g + 1$ is also a profitable lie for g.

For each $j \in R \setminus \{g\}$, let k_j be the sum of truthful positive signals received by j (excluding group g's information) and m_j be the total number of truthful signals observed by j, again excluding g's information. If it is profitable for g to lie with $\tilde{s}$ it must be that:

$$- \sum_{j \in R \setminus \{g\}} \alpha_j^R \left(\beta_j + E(\theta | k_j + \tilde{s}, m_j + n_g) - \beta_g - E(\theta | k_j + s_g, m_j + n_g)\right)^2 + \sum_{j \in R \setminus \{g\}} \alpha_j^R \left(\beta_j - \beta_g\right)^2 > 0.$$

Expanding and collecting terms, this implies:

$$\sum_{j \in R \setminus \{g\}} \alpha_j^R \left(E(\theta | k_j + \tilde{s}, m_j + n_g) - E(\theta | k_j + s_g, m_j + n_g)\right) \cdot$$

$$\left(2\beta_j - 2\beta_g + E(\theta | k_j + \tilde{s}, m_j + n_g) - E(\theta | k_j + s_g, m_j + n_g)\right) < 0.$$

Plugging in the values for $E(\theta | \cdot)$, we get that:

$$\sum_{j \in R \setminus \{g\}} \alpha_j^R \left(\frac{\tilde{s} - s_g}{m_j + n_g + 2}\right) \left(2\beta_j - 2\beta_g + \frac{\tilde{s} - s_g}{m_j + n_g + 2}\right) < 0,$$

and dividing both sides by $\tilde{s} - s_g$, which we can do because $\tilde{s} - s_g > 0$, we get:

$$\sum_{j \in R \setminus \{g\}} \left(\frac{\alpha_j^R}{m_j + n_g + 2} \right) \left(2\beta_j - 2\beta_g + \frac{\tilde{s} - s_g}{m_j + n_g + 2} \right) < 0.$$

Since we assumed that $\tilde{s} > s_g$, it follows that the term $\frac{\tilde{s} - s_g}{m_j + n_g + 2} > 0$ for all j. This implies that if the above condition holds for some $\tilde{s} > s_g + 1$, then it also holds for $s_g + 1$. A similar argument holds if we had instead assumed that $\tilde{s} < s_g - 1$ represented a profitable deviation for g. I omit this half of the proof. $\square$

Proof of Condition 1.

The proof is similar to that of Galeotti et al. (2013). Let $E_{-g} = E \setminus \{g\}$, $D_{-g} = D \setminus \{g\}$. In the same fashion, let $s_{E_{-g}} = \sum_{h \in E_{-g}} s_h$, the sum of positive signals received by the groups in E_{-g}, and let $n_{E_{-g}}$ be the total number of signals received by the groups in E_{-g}. Let R be an association, with $h \in E \subseteq R$ being the groups that truthfully reveal their signals and $j \in D \subseteq R$ being those that babble. As shown in Lemma 1, if g is incentivized to communicate truthfully, then g receives no benefit from misrepresenting the signal of a single one of its members. For a group $h \in E$ the total observed number of (true) positive signals is s_E and for a group $j \in D$, the total observed number of true positive signals is $s_E + s_j$, as j observes the signals of individuals in all groups that have chosen to engage, and also the private signals of its own members.

Group g's expected payoff to truthful communication to an association consisting of itself and $\{E_{-g}, D_{-g}\}$ is

$$- \sum_{h \in E_{-g}} \left[\alpha_h^R \left(y_h^*(s_{E_{-g}} + s_g, n_{E_{-g}} + n_g) - E(\theta | s_{E_{-g}} + s_g, n_{E_{-g}} + n_g) - \beta_g \right)^2 \right]$$

$$- \sum_{j \in D} \left[\alpha_j^R \left(y_j^*(s_{E_{-g}} + s_g + s_j, n_{E_{-g}} + n_g + n_j) - E(\theta | s_{E_{-g}} + s_g + s_j, n_{E_{-g}} + n_g + n_j) - \beta_g \right)^2 \right] \quad (8)$$

and group $g's$ payoff to misrepresenting the signal of a single one of its members is

40

$$- \sum_{h \in E_{-g}} \left[\alpha_h^R \left(y_h^*(s_{E_{-g}} + s_g \pm 1, n_{E_{-g}} + n_g) - E(\theta | s_{E_{-g}} + s_g, n_{E_{-g}} + n_g) - \beta_g \right)^2 \right]$$

$$- \sum_{j \in D} \left[\alpha_j^R \left(y_j^*(s_{E_{-g}} + s_g + s_j \pm 1, n_{E_{-g}} + n_g + n_j) - E(\theta | s_{E_{-g}} + s_g + s_j, n_{E_{-g}} + n_g + n_j) - \beta_g \right)^2 \right],$$

(9)

where the $\pm$ term represents whether g misrepresented a negative signal as positive or vice versa. Therefore g has an incentive to truthfully report the positive signals of its members if and only if Equation 8 $\geq$ Equation 9.

To simplify notation, for the remainder of the proof I will let $\mathbf{p_j}$ be the collection of truthful positive signals observed by group j along with g's (true) positive signals and $\mathbf{n_j}$ be the total number of truthful signals observed by group j along with g's signals. Thus, for $h \in E_{-g}$ we have $\mathbf{p_h} = s_{E_{-h}} + s_g$ and $\mathbf{n_h} = n_{E_{-g}} + n_g$. For $j \in D_{-h}$ we have $\mathbf{p_j} = s_{E_{-g}} + s_j + s_g$, and so on. Reducing, and using the fact that $(a + b)(a - b) = a^2 - b^2$, the necessary and sufficient condition for truthful communication by group g to association R is

$$- \sum_{i \in R \setminus \{g\}} \alpha_i^R (y_i^*(\mathbf{p_i}, \mathbf{n_i}) - y_i^*(\mathbf{p_i} \pm 1, \mathbf{n_i}))(y_i(\mathbf{p_i}, \mathbf{n_i}) + y_i(\mathbf{p_i} \pm 1, \mathbf{n_i}) - 2E(\theta | \mathbf{p_i}, \mathbf{n_i}) - 2\beta_g) \geq 0.$$

Dividing both sides by 2 and letting y_i^* be defined as in Equation 5, we get

$$- \sum_{i \in R \setminus \{g\}} \alpha_i^R \left(\beta_i + \frac{\mathbf{p_i} + 1}{\mathbf{n_i} + 2} - \beta_i - \frac{\mathbf{p_i} + 1 \pm 1}{\mathbf{n_i} + 2} \right) \left(\frac{2\beta_i + \frac{\mathbf{p_i} + 1}{\mathbf{n_i} + 2} + \frac{\mathbf{p_i} + 1 \pm 1}{\mathbf{n_i} + 2}}{2} - \frac{\mathbf{p_i} + 1}{\mathbf{n_i} + 2} - \beta_g \right) \geq 0.$$

Further reducing and letting $\tilde{s}_g$ be the single signal that g potentially wishes to lie about we get

$$- \sum_{i \in R \setminus \{g\}} \alpha_i^R \left(\frac{-1}{\mathbf{n_i} + 2} \right) \left(\frac{1}{2(\mathbf{n_i} + 2)} + \beta_i - \beta_g \right) \geq 0 \quad \text{if} \quad \tilde{s}_g = 0$$

$$- \sum_{i \in R \setminus \{g\}} \alpha_i^R \left(\frac{1}{\mathbf{n_i} + 2} \right) \left(\frac{-1}{2(\mathbf{n_i} + 2)} + \beta_i - \beta_g \right) \geq 0 \quad \text{if} \quad \tilde{s}_g = 1,$$

which can be rewritten as

41

$$\sum_{i\in R\setminus\{g\}} \alpha_i^R(\tfrac{1}{\mathbf{n_i}+2})(\tfrac{1}{2(\mathbf{n_i}+2)}) \geq -\sum_{i\in R\setminus\{g\}} \alpha_i^R(\tfrac{1}{\mathbf{n_i}+2})(\beta_i-\beta_g) \quad \text{if} \quad \tilde{s}_g = 0$$

$$\sum_{i\in R\setminus\{g\}} \alpha_i(\tfrac{1}{\mathbf{n_i}+2})(\tfrac{1}{2(\mathbf{n_i}+2)}) \geq \sum_{i\in R\setminus\{g\}} \alpha_i^R(\tfrac{1}{\mathbf{n_i}+2})(\beta_i-\beta_g) \quad \text{if} \quad \tilde{s}_g = 1.$$

Last, the above can be combined into a single inequality:

$$\sum_{i\in R\setminus\{g\}} \frac{\alpha_i^R}{2(\mathbf{n_i}+2)^2} \geq \left| \sum_{i\in R\setminus\{g\}} \frac{\alpha_i^R}{\mathbf{n_i}+2}(\beta_i-\beta_g) \right|.$$

Substituting in the values for $\mathbf{p_i}$ and $\mathbf{n_i}$ for $j \in D_{-g}$ and $h \in E_{-g}$ yields the statement of this condition. $\square$

Calculations for example in Section 5.

First, to see that there is no vector α that can induce truthful messaging by groups 1 or 2 note that truthful communication for 1 requires that

$$\frac{\alpha_2}{m_2+4}\left(\frac{1}{2(m_2+4)}-.07\right) + \frac{\alpha_3}{m_3+4}\left(\frac{1}{2(m_3+4)}-.17\right) \geq 0,$$

and truthful communication for 3 requires

$$\frac{\alpha_1}{m_1+3}\left(\frac{1}{2(m_1+3)}-.17\right) + \frac{\alpha_2}{m_2+3}\left(\frac{1}{2(m_2+3)}-.1\right) \geq 0.$$

The incentive is easiest to satisfy for each group when the m_i (number of signals received by outgroup i) are minimized, which occurs at $m_i = n_i$; both inequalities fail at this point.

Second, 2's association condition requires:

$$-\alpha_1(.07)^2 - \alpha_3(.1)^2 - \alpha_1(\frac{1}{48}) - \alpha_3(\frac{1}{42}) - (1-\alpha_1-\alpha_3)(\frac{1}{36}) + \frac{1}{36} \geq 0.$$

This equation reduces to the requirement that $\alpha_1 \geq 2.95\alpha_3$. Similarly, satisfaction of truthful communication for 2 additionally requires

$$\frac{\alpha_1}{128} + \frac{\alpha_3}{98} - \left| \frac{.07\alpha_1}{8} - \frac{.1\alpha_3}{7} \right| \geq 0.$$

This condition reduces to the pair of inequalities $.25\alpha_3 \leq \alpha_1 \leq 26.12\alpha_3$.

Calculations for example in Section 6.

Let $\beta_1 = 0, \beta_2 > 0$ and $n_1 = n_2 = 7$. A pure strategy for each group is a mapping $\rho_g :$ $\{0, 1, ..., n_g\} \times 2^{|G|} \rightarrow \{0, 1, 2, ..., n_g\}$ from the set of types g could be, and the set of groups in its association, into a public message, m_g. If group g plays a semi-separating strategy then the range of ρ_g has strictly more than one element in it and strictly less than $n_g + 1$: some types that g could be are mapped into the same message.

Suppose group 1 observes a number of positive signals s_1 and 2's message m_2. Then by sequential rationality 1 chooses y_1^* by calculating the probability that 2 is each of the types mapped into m_2. At the same time, 2's incentive to deviate from messaging strategy m_2 is calculated by computing the expected value of y_1^* conditional on 2 sending correct message m_2 versus some different m_2', and conditional on 2 having received s_2 positive signals.

Recall that if m total signals are received, the probability that $k = t$ are positive is

$$Pr[k = t|m] = \int_{\theta=0}^{1} \binom{m}{t}\theta^t(1 - \theta)^{m-t}d\theta = \frac{1}{m + 1}.$$

If m total signals are received and k are positive, the posterior distribution of θ is

$$f(\theta|k, m) = (m + 1)\binom{m}{k}\theta^k(1 - \theta)^{m-k}.$$

Therefore if 1 has received n_1 signals and s_1 signals were positive, its posterior belief that 2 has received $s_2 = k$ positive signals is:

$$Pr[s_2 = k|s_1] = \int_{\theta=0}^{1} \binom{n_2}{k}\theta^k(1 - \theta)^{n_2-k}f(\theta|s_1, n_1)d\theta.$$

These posterior beliefs for $n_1 = n_2 = 7$ are shown in the table below, with the columns representing the positive signals one group has observed and the rows representing the likelihood the other group received that number of positive signals.

Posterior	Observed positives							
	0	1	2	3	4	5	6	7
0	$\frac{8}{15}$	$\frac{4}{15}$	$\frac{8}{65}$	$\frac{2}{39}$	$\frac{8}{429}$	$\frac{4}{715}$	$\frac{8}{6435}$	$\frac{1}{6435}$
1	$\frac{4}{15}$	$\frac{56}{195}$	$\frac{14}{65}$	$\frac{56}{429}$	$\frac{28}{429}$	$\frac{56}{2145}$	$\frac{49}{6435}$	$\frac{8}{6435}$
2	$\frac{8}{65}$	$\frac{14}{65}$	$\frac{168}{715}$	$\frac{28}{143}$	$\frac{56}{429}$	$\frac{49}{715}$	$\frac{56}{2145}$	$\frac{4}{715}$
3	$\frac{2}{39}$	$\frac{56}{429}$	$\frac{28}{143}$	$\frac{280}{1287}$	$\frac{245}{1287}$	$\frac{56}{429}$	$\frac{28}{429}$	$\frac{8}{429}$
4	$\frac{8}{429}$	$\frac{28}{429}$	$\frac{56}{429}$	$\frac{245}{1287}$	$\frac{280}{1287}$	$\frac{28}{143}$	$\frac{56}{429}$	$\frac{2}{39}$
5	$\frac{4}{715}$	$\frac{56}{2145}$	$\frac{49}{715}$	$\frac{56}{429}$	$\frac{28}{143}$	$\frac{168}{715}$	$\frac{14}{65}$	$\frac{8}{65}$
6	$\frac{8}{6435}$	$\frac{49}{6435}$	$\frac{56}{2145}$	$\frac{28}{429}$	$\frac{56}{429}$	$\frac{14}{65}$	$\frac{56}{195}$	$\frac{4}{15}$
7	$\frac{1}{6435}$	$\frac{8}{6435}$	$\frac{4}{715}$	$\frac{8}{429}$	$\frac{2}{39}$	$\frac{8}{65}$	$\frac{4}{15}$	$\frac{8}{15}$

Since truthful messaging is not satisfied for either group in this example, an asymmetric semi-separating equilibrium where one group is truthful and the other is partially informative does not exist; the condition will be broken for the truthful group. Therefore any semi-separating equilibrium involves a non-trivial bundling of types for both groups. Moreover, the bundling cannot involve two consecutive types being in singleton sets, because this offers a player the ability to misrepresent a single signal; in this setting the incentive to misrepresent a single signal is independent of the number of signals actually received. An example (which we can computationally show is the best partitional equilibrium) is the following mapping:[20]

$$\rho_2(0) = \rho_2(1) = m_2^1; \rho_2(2) = m_2^2; \rho_2(3) = \rho_2(4) = m_2^3; \rho_2(5) = m_2^4; \rho_2(6) = \rho_2(7) = m_2^5. \quad (10)$$

[20]The mapping represents a slight abuse of notation as it disregards the R element of the domain of ρ_2; since there are only two groups (and thus one possible association) it is unnecessary.

Given that 2 is playing ρ_2, 1 calculates $y_1^*(m_2|s_1)$ as follows:

$$y_1^*(m_2|s_1) = \sum_{k:\rho(k)=m_2} \frac{Pr[s_2 = k|s_1] * E(\theta|k + s_1, n_1 + n_2)}{\sum_{j:\rho(j)=m_2} Pr[s_2 = j|s_1]}. \tag{11}$$

Thus, conditional on each type 2 could be and message 2 could send if playing ρ_2, 2 calculates the expected value of $y_1^*(m_2)$ as

$$E(y_1^*(m_2)|s_2) = \sum_{k=0}^{n_1} y_1^*(m_2|s_1) * Pr[s_1 = k|s_2].$$

Using the table above to calculate $Pr[s_g = k|s_h]$, 2's truthful messaging conditions are calculated for each s_2 and m_2 by evaluating the maximum difference in bias that that the two groups could have for which 2 has no incentive to deviate from ρ_2. For the example of ρ_2 described in Equation 10, truthful messaging holds for 2 for $\beta_2 \leq .0408$. When $\beta_2 = .033$, letting $\rho_1 = \rho_2$ as described in Equation 10 can be shown to represent the best partitional equilibrium for both groups. Letting $y_2^*(m_1|s_2)$ be calculated analogously to y_1^* in Equation 11, the expected utility each group is expected to receive is calculated as:

$$EU_1(y^*|\rho_1, \rho_2) =$$
$$\sum_{k=0}^{n_1}\sum_{j=0}^{n_2} \frac{Pr[s_2 = j|s_1 = k]}{n_1 + 1} \int_{\theta=0}^{1} \left(-.5(y_1^*(m_2|s_1) - \theta)^2 - .5(y_2^*(m_1|s_2) - \theta)^2\right) f(\theta|k + j, n_1 + n_2)d\theta$$

and

$$EU_2(y^*|\rho_1, \rho_2) =$$
$$\sum_{k=0}^{n_1}\sum_{j=0}^{n_2} \frac{Pr[s_2 = j|s_1 = k]}{n_1 + 1} \int_{\theta=0}^{1} \left(-.5(y_1^*(m_2|s_1) - \theta - \beta_2)^2 - .5(y_2^*(m_1|s_2) - \theta - \beta_2)^2\right) f(\theta|k+j, n_1+n_2)d\theta.$$

By the symmetry of the problem these values are identical, and equal $-.011634$.

References

John Adams. A dissertation on the canon and feudal law. *The Works of John Adams*, 3:447–452, 1765.

David Austen-Smith and Timothy Feddersen. *Deliberation and voting rules*. In *Social Choice and Strategic Decisions: Essays in honor of Jeffrey S. Banks*. Springer, 2005. David Austen-Smith and John Duggan, ed.

David Austen-Smith and Timothy J Feddersen. Deliberation, preference uncertainty, and voting rules. *American Political Science Review*, 100(2):209, 2006.

Mariagiovanna Baccara and Leeat Yariv. Homophily in peer groups. *American Economic Journal: Microeconomics*, 5(3):1–30, 2013.

Joshua Cohen. Deliberation and democratic legitimacy. *Debates in Contemporary Political Philosophy*, pages 342–360, 1989.

Marquis de Condorcet. Essay on the application of analysis to the probability of majority decisions. *Paris: Imprimerie Royale*, 1785.

Vincent P Crawford and Joel Sobel. Strategic information transmission. *Econometrica: Journal of the Econometric Society*, pages 1431–1451, 1982.

Jacques Cremer and Thomas R Palfrey. Political confederation. *American Political Science Review*, pages 69–83, 1999.

Torun Dewan and Francesco Squintani. The role of party factions: An information aggregation approach. *Working Paper, University of Warwick*, 2012.

Torun Dewan, Andrea Galeotti, Christian Ghiglino, and Francesco Squintani. Information aggregation and optimal structure of the executive. *Mimeo*, 2011.

Eric S Dickson, Catherine Hafer, and Dimitri Landa. Cognition and strategy: a deliberation experiment. *The Journal of Politics*, 70(04):974–989, 2008.

Jon X Eguia. Discrimination and assimilation. *Mimeo*, 2012.

Sean Gailmard and John W Patty. Giving advice vs. making decisions: Transparency, information, and delegation. *Working paper, Washington University in Saint Louis*, 2013.

Andrea Galeotti, Christian Ghiglino, and Francesco Squintani. Strategic information transmission networks. *Journal of Economic Theory*, 2013.

Dino Gerardi and Leeat Yariv. Deliberative voting. *Journal of Economic Theory*, 134(1):317–338, 2007.

Catherine Hafer and Dimitri Landa. Deliberation as self-discovery and institutions for political speech. *Journal of Theoretical Politics*, 19(3):329–360, 2007.

FA Hayek. The use of knowledge in society. *The American Economic Review*, 35(4):519–530, 1945.

Albert O Hirschman. *Exit, voice, and loyalty: Responses to decline in firms, organizations, and states*, volume 25. Harvard University Press, 1970.

Matias Iaryczower and Santiago Oliveros. Power brokers: Middlemen in legislative bargaining. *Available at SSRN 2236557*, 2013.

Dimitri Landa and Adam Meirowitz. Game theory, information, and deliberative democracy. *American Journal of Political Science*, 53(2):427–444, 2009.

Bernard Manin, Elly Stein, and Jane Mansbridge. On legitimacy and political deliberation. *Political theory*, 15(3):338–368, 1987.

Adam Meirowitz. In defense of exclusionary deliberation: communication and voting with private beliefs and values. *Journal of Theoretical Politics*, 19(3):301–327, 2007.

William Minozzi, Michael A. Neblo, and David A. Siegel. Deliberation as interactive reasoning. *Mimeo*, 2013.

John Morgan and Phillip C Stocken. Information aggregation in polls. *The American Economic Review*, pages 864–896, 2008.

Josiah Ober. *Democracy and knowledge: Innovation and learning in classical Athens*. Princeton University Press, 2008.

Scott E Page. *The Difference: How the Power of Diversity Creates Better Groups, Firms, Schools, and Societies (New Edition)*. Princeton University Press, 2008.

John W Patty. A theory of cabinet-making: The politics of inclusion, exclusion, and information. *Working paper, Washington University in Saint Louis*, 2013.

John W. Patty and Elizabeth Maggie Penn. Sequential decision-making & information aggregation in small networks. *Forthcoming, Political Science Research and Methods*, 2013.

Keith Schnakenberg. Group identity and symbolic political behavior. *Working paper, Washington University in Saint Louis*, 2013.

James Surowiecki. *The wisdom of crowds*. Random House Digital, Inc., 2005.

Iris Marion Young. *Inclusion and Democracy*. Oxford University Press, 2002.