

Politics Transformed? How Ranked Choice Voting Shapes Electoral Strategies

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Abstract

We compare electoral outcomes under plurality rule versus ranked choice voting (RCV). Candidates compete by choosing platforms that can either mobilize their core supporters, or instead attract undecided voters. RCV exacerbates platform polarization in contexts of low voter engagement, strong partisan attachments, and imbalances in the candidates' share of core supporters. RCV may increase or decrease voter turnout relative to plurality rule, and strong partisan attachments increase the likelihood that the winning candidate receives a minority of votes cast.

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1. Introduction

Ranked Choice Voting (RCV) is the most publicly-debated and rapidly-expanding electoral reform in the United States. Rather than voting for a single candidate, voters under RCV rank multiple candidates.¹ If a candidate wins a majority of first preferences, she is elected. If no candidate wins a majority of first preferences, the candidate with the fewest first preferences is eliminated, and each of her ballots transfers to the next-ranked candidate. The process repeats until a single candidate wins a majority of the remaining ballots.

RCV is widely employed in local and state elections, both in general elections and also in the primaries of both major US political parties.² A notable recent example is New York City, which adopted RCV in its primary elections for both Mayor and City Council in 2019. The change was endorsed by a broad coalition of political actors, and hailed as “a smart, tested reform that would make certain that New Yorkers elect candidates who have the support of a majority of voters” ([New York Times 2019](#)).

Existing work shows how RCV shapes candidate and voter behavior—for example, by spurring candidate entry ([Callander 2005](#); [Dellis, Gauthier-Belzile and Oak 2017](#)), or attenuating incentives to vote strategically ([Eggers and Nowacki 2021](#)). Our paper assesses a number of widely-held contentions about RCV: most importantly, that it encourages candidates to pursue ideologically moderate policy agendas in order to broaden their electoral support.³ This contention is advanced by both scholars ([Horowitz 2004](#); [Drutman 2020](#)) and electoral reform advocates including *FairVote* in the United States and the United Kingdom’s *Electoral Reform Society*.

The contention rests on the following logic. Under plurality, a candidate only benefits from

¹While variants of Ranked Choice Voting can also be used in multi-member districts (for a review, see [Santucci 2021](#)), in this paper we focus on the more common version with single-member districts—also called Instant Runoff. In this paper, we use RCV as a synonym of Instant Runoff, though the latter is technically a special case of the former.

²For a comprehensive list, see <https://www.fairvote.org/>.

³For a summary of RCV’s proposed benefits, see [Cormack \(2021\)](#).

the support of voters that prefer her to every other candidate. This encourages a candidate to focus her policy appeal on mobilizing the narrow segment of voters that are most likely to prefer her over all other candidates—usually, her ideological or ethnic base. Under RCV, by contrast, a candidate can benefit from the support of voters that do not like her the most. The prospect of winning these voters' second preferences raises a candidate's relative benefit from broadening her policy appeal to attract support from these voters, instead of focusing exclusively on her base.

Our paper asks: is this contention correct? When does RCV provide greater incentives for candidates to moderate their policy platforms than plurality rule? Does RCV necessarily increase voter participation? And, under what conditions does RCV lead to the election of a candidate with the support of a majority of voters?

To address these questions, we develop a model of electoral competition between three office-seeking candidates: A , B and C . Each candidate has a group of core supporters (her 'base') and there is also a group of moderate (or 'swing') voters whose relative preference for A versus B is uncertain. C 's base is the largest; together, the voters in the remaining groups constitute a majority, but their support is divided between A and B . Ours is therefore a classical 'divided majority' setting with aggregate uncertainty (e.g., [Myatt 2007](#); [Bouton and Castanheira 2012](#); [Bouton, Castanheira and Llorente-Saguer 2016](#)). Without loss of generality, A 's share of core supporters exceeds B 's: A is the *advantaged* majority candidate, while B is the *disadvantaged* majority candidate.

Candidates A and B simultaneously choose platforms: minority-preferred C does not play a strategic role, but her presence implies that competition between the remaining candidates is non-zero-sum. After candidates choose platforms, moderate voters realize an aggregate preference shock in favor of A versus B . All voters then choose whether to cast a ballot, or abstain. Under plurality, a voter that turns out casts a ballot for a single candidate; under RCV, a voter that turns out also chooses whether to express a preference for one or more candidates.

Voting decisions are guided by a simple heuristic based on *abstention due to alienation*, developed theoretically in [Hinich, Ledyard and Ordeshook \(1972\)](#), [Callander and Wilson \(2007\)](#),

and [Zakharov \(2008\)](#). In this heuristic, a voter always ranks candidates according to her sincere preference, but she only awards a preference to candidates whose platforms give her a sufficiently high payoff. Implied turnout patterns find empirical support in plurality rule elections ([Adams, Dow and Merrill 2006](#), [Stewart III, Alvarez, Pettigrew and Wimpy 2020](#)). Our approach further implies that a voter who turns out under RCV may not fully utilize her ballot, a widely-documented phenomenon termed *ballot exhaustion* ([Burnett and Kogan 2015](#)).⁴

Our core analysis studies each majority candidate A 's and B 's choice of whether to direct her policy appeals towards her base (a 'base' strategy), or instead to attract moderate voters (a 'swing' strategy). Targeting the base boosts their enthusiasm, and thus raises their turnout. Targeting moderates increases the probability that they rank the candidate first and, in that event, it also raises their turnout for the candidate. On the one hand, each candidate's expected return from targeting moderates is reduced by their uncertainty about moderates' preferences. On the other hand, if there are enough moderate voters, their support may be decisive for the election.

We obtain a unique equilibrium under both plurality and RCV. Because C 's base is the largest, she is the frontrunner under plurality rule. A and B therefore both seek to mobilize enough first preferences from their own base and from moderates to defeat C . Turning out voters in either group contributes votes in proportion to that group's size. Each candidate's equilibrium strategy therefore resolves in favor of appealing to moderates if and only if their share within the majority exceeds the corresponding share of that candidate's base.

A candidate's strategic calculus is quite different under RCV. Either majority candidate A or B wins the election if she secures enough first preferences to defeat the other majority candidate, *and* if her combined first and second preferences are sufficient to defeat C . If ad-

⁴Our approach to voting behavior contrasts with formulations in which voters condition their choice on the relative prospects of pivotal events. The strategic and computational burden such behavior imposes on voters leads scholars to question its plausibility in real-world plurality rule elections ([Van der Straeten, Laslier, Sauger and Blais 2010](#)). This burden intensifies under RCV: in a three-candidate contest, the set of pivotal events expands from three under plurality rule to twelve under RCV ([Eggers and Nowacki 2021](#)).

vantaged A wins more first preferences than disadvantaged B , she also picks up additional support from moderates and B 's core supporters that give A their second preferences. These second preferences may be sufficient for A to defeat the plurality candidate C . In these contexts A 's focus on defeating C under plurality shifts to defeating B under RCV.

Whether a base strategy or a swing strategy best achieves A 's goal depends on the relative size of each of each of the groups, the intensity of core supporters' partisan attachments, and voters' overall engagement—i.e., their propensity to abstain due to alienation. We unearth contexts in which advantaged A is *more* inclined to target her base under RCV, versus plurality rule. The contexts arise when the imbalance in each candidate's share of core supporters is relatively large, when core supporters' partisan attachments are strong, and when average voter engagement is low. These factors all tilt the pairwise contest between A and B in favor of the candidate whose base turns out the most on election day.

Matters are very different for the disadvantaged majority candidate B , whose incentives to moderate are *always* stronger under RCV than under plurality rule. In this case, moderation helps the weaker candidate buttress her smaller share of first preference support with additional second preferences from moderate voters, as well as A 's core supporters. These second preferences are valuable in her efforts to defeat C .

So, relative to plurality rule, RCV raises an advantaged candidate's incentives to moderate when her advantage over her opponent isn't too large; otherwise, RCV leads to even less compromise and *more* policy extremism. By contrast, RCV always strengthens the disadvantaged candidate's incentive to moderate.

In addition, we show that RCV may increase or decrease turnout relative to plurality. In particular, when the bases of the majority candidates A and B are large enough, turnout under RCV is lower than under plurality. And, strong partisan attachment amongst core voters raises the possibility that the winning candidate under RCV may fail to achieve a majority of the total ballots cast.

Related Literature. [Grofman and Feld \(2004\)](#) identify contexts in which RCV improves the likelihood of electing a Condorcet winner when one exists, relative to plurality rule. They

posit sincere voting, but restrict their analysis to a fixed set of policies. Two papers study policy outcomes under RCV in a spatial model of elections: [Callander \(2005\)](#) and [Dellis, Gauthier-Belzile and Oak \(2017\)](#). Both derive equilibria with two candidates who locate at polarized platforms: polarization is bounded by the threat of another candidate contesting the election at a centrist position between the two platforms.

In a citizen-candidate framework, [Dellis, Gauthier-Belzile and Oak \(2017\)](#) show that this bound is tightest under RCV. The reason is that a centrist entrant wins so long as she doesn't receive the *least* first preferences, since her centrist platform wins every voter's second preference. Under plurality, by contrast, a centrist entrant wins only if she receives the *most* first preferences. The authors conclude that RCV sustains less policy polarization than the plurality rule. [Callander \(2005\)](#) characterizes a continuum of equilibria in his office-motivated Downsian framework, highlighting the co-existence of equilibria with full median convergence under RCV, alongside equilibria with polarized platforms. Under plurality, by contrast, median convergence with three or more candidates cannot be supported ([Cox 1987](#)).

Because all voters turnout and fully utilize their ballots in both papers, and because candidates are differentiated solely by platforms, these frameworks do not address how candidates use their policy commitments to mobilize core supporters versus moderates. While our three-candidate framework abstracts from the question of how *many* candidates can be supported under RCV, both [Callander \(2005\)](#)'s and [Dellis, Gauthier-Belzile and Oak \(2017\)](#)'s analysis with endogenous candidacy highlights the stability of three-or-fewer candidate competition under RCV. This is also consistent with evidence from real-world elections, documented in [Jesse \(2000\)](#) and [Farrell and McAllister \(2006\)](#).

2. Model

Players. Three candidates A , B and C compete for the support of a mass of voters whose size we normalize to $1 + \gamma$. Only candidates A and B are strategically active, and a minority $\gamma < 1$ of voters only cast ballots for C . Each individual in the remaining unit mass of voters (i.e., the majority) belongs to a group $j \in \{-1, 0, 1\} \equiv X$. A share α in the majority belong to group -1 ,

a share β belong to group 1, and the remaining $1 - \alpha - \beta$ majority voters belong to group 0.

Preferences. Candidates are office-seeking. If candidate $i \in \{A, B\}$ wins the election and implements policy $y_i \in X$, a group- j voter's payoff is

$$u_j^A(y_A; \tau_j) = -|x_j - y_A| + \tau_j \quad (1)$$

$$u_j^B(y_B; \tau_j) = -|x_j - y_B| - \tau_j \quad (2)$$

Voters derive linear losses in the distance between the policy outcome and their preferred policy. We associate each voter's ideal point x_j with her group identity, $j \in \{-1, 0, 1\}$. Voters further derive a benefit τ_j from candidate A regardless of her platform x_j . This benefit could reflect a fixed policy position on another dimension of issue conflict amongst the majority. We assume $\tau_{-1} = \theta > 0$, and $\tau_1 = -\theta < 0$, so that voters in group -1 are candidate A 's *base*, and voters in group 1 are candidate B 's base. Higher values of θ correspond to stronger partisan attachments: core supporters increasingly favor their own candidate and dislike the other candidate. The preferences of voters in group 0 are uncertain: τ_0 is drawn from a symmetric cumulative distribution G with support $[-\theta, \theta]$. We therefore call group-0 voters *moderates* or *centrists*. We implicitly assume that voters within the majority always prefer either candidate A or B to candidate C .

Voters also care about their participation in elections. Every voter has an idiosyncratic *reservation utility* ρ and votes for her most-preferred candidate if and only if this candidate's value—the greatest of (1) and (2)—exceeds ρ . Under RCV, she further casts a preference vote for any other candidate(s) whose value exceeds ρ . We assume that ρ is uniformly distributed on the interval $[\bar{\rho} - 1/(2\phi), \bar{\rho} + 1/(2\phi)]$; $F(x)$ denotes its cumulative distribution. Higher values of $\bar{\rho}$ can be interpreted as lower *voter engagement*—possibly due to limited information or enthusiasm about the candidates.

Timing. The interaction proceeds as follows.

1. Candidates A and B simultaneously select platforms $(y_A, y_B) \in \{-1, 0, 1\}^2$.
2. The preference shock τ_0 is realized, and each voter makes her voting decision.
3. The winning candidate implements her promised platform, and payoffs are realized.

Since we interpret the fraction γ voters that only cast ballots for non-strategic C as that candidate's base, we implicitly assume C locates at that group's preferred policy, and thus C 's vote share is always $\gamma F(\theta)$ under both plurality and RCV.

Equilibrium. We study Nash equilibria.

Assumptions. Without loss of generality, we assume A 's base exceeds B 's: $\alpha \geq \beta$. For that reason, we call A the *advantaged* majority candidate, and B the *disadvantaged* majority candidate. We also assume $\theta > 2$, ensuring that each candidate is always most-preferred by the voters in her base. We assume that ρ 's support is large enough to ensure (i) non-zero turnout amongst all groups, and (ii) that both candidates have an interior probability of winning. This holds if (i) $\min \left\{ \frac{1}{2\phi} - \bar{\rho}, \frac{1}{2\phi} + \bar{\rho} \right\} > \theta + 2$ and (ii) $\phi < \frac{1-\alpha-\gamma}{2}$. Finally, we assume that the majority is sufficiently divided between its two candidates A and B that group C is the frontrunner in a plurality context. Formally, we assume

$$\max \{ (1 - \alpha - \beta)F(0) + \alpha F(\theta - 1), (1 - \alpha - \beta)F(-1/2) + \alpha F(\theta) \} < \gamma F(\theta), \quad (3)$$

which amounts to assuming that the mass γ of C 's core supporters is large—for example, it implies that they outnumber A 's (mass α) and B 's (mass β). Our framework does not require on this assumption, but it is the most interesting context because it creates the greatest scope for differences in behavior across the systems.

3. Equilibrium under Plurality Rule

Under plurality rule, voters can express a single preference for their most-preferred candidate. A group- j voter turns out if and only if her most-preferred candidate $i \in \{A, B\}$'s payoff $u_j^i(y_i; \tau_j)$ exceeds her idiosyncratic reservation utility, ρ . In other words, she votes for her most-preferred candidate if

$$\max \{ u_j^A(y_A; \tau_j), u_j^B(y_B; \tau_j) \} \geq \rho. \quad (4)$$

If the inequality (4) is instead reversed, the voter abstains. Recalling that $F(\cdot)$ denotes ρ 's cumulative distribution, the share of voters in group j that turn out for their preferred candidate i is therefore $F(u_j^i(y_i; \tau_j))$. Letting $\pi(j)$ denote the share of voters in the majority belonging to each group $j \in \{-1, 0, 1\}$, and $\mathbf{1}\{\cdot\}$ the indicator function, we obtain candidate A 's total first preferences—and thus her total votes under plurality rule.

$$v_A^f(y_A, y_B; \tau_0) \equiv \sum_{j \in \{-1, 0, 1\}} \pi(j) F(u_j^A(y_A; \tau_j)) \mathbf{1}\{u_j^A(y_A; \tau_j) \geq u_j^B(y_B; \tau_j)\}. \quad (5)$$

Likewise, B 's total first preferences and thus her total votes under plurality rule is:

$$v_B^f(y_A, y_B; \tau_0) \equiv \sum_{j \in \{-1, 0, 1\}} \pi(j) F(u_j^B(y_A; \tau_j)) \mathbf{1}\{u_j^B(y_A; \tau_j) < u_j^B(y_B; \tau_j)\}. \quad (6)$$

Our assumption $\theta > 2$ implies that A is always preferred by her core supporters in group -1 . By targeting these voters with a policy $y_A = -1$, A raises their enthusiasm, and thus their turnout. Similarly, voters B 's core supporters in group 1 always prefer her. In contrast, the preferences of centrist group -0 voters are uncertain because of the aggregate preference shock, τ_0 . These voters prefer A if and only if $u_0^A(y_A; \tau_0) \geq u_0^B(y_B; \tau_0)$, which is equivalent to:

$$-|y_A - 0| + \tau_0 \geq -|y_B - 0| - \tau_0 \iff \tau_0 \geq \frac{|y_A| - |y_B|}{2} \equiv \hat{\tau}_0(y_A, y_B).$$

If A targets her policy appeal to centrists by locating at $y_A = 0$, she lowers the threshold $\hat{\tau}_0$ to be their most-preferred candidate, and further raises their enthusiasm and thus their turnout. But even if A targets these voters, they may nonetheless prefer B , which occurs if $\tau < \hat{\tau}_0$.

We conclude that A 's total votes are:

$$\begin{aligned} v_A^f(y_A, y_B; \tau_0) &= \alpha F(u_{-1}^A(y_A; \theta)) + (1 - \alpha - \beta) \mathbf{1}\{\tau_0 > \hat{\tau}_0\} F(u_0^A(y_A; \tau_0)) \\ &= \alpha F(\theta - 1 - y_A) + (1 - \alpha - \beta) \mathbf{1}\{\tau_0 > \hat{\tau}_0\} F(\tau_0 - |y_A|). \end{aligned} \quad (7)$$

Similarly, B wins total votes

$$v_B^f(y_A, y_B; \tau_0) = \beta F(\theta - 1 + y_B) + (1 - \alpha - \beta) \mathbf{1}\{\tau_0 < \hat{\tau}_0\} F(-\tau_0 - |y_B|). \quad (8)$$

Our benchmark result characterizes the unique equilibrium under plurality rule, which depends on the imbalance in size between the candidates' bases.

Proposition 1. *There exists a (generically) unique equilibrium under plurality:*

- (i) if $\alpha < \alpha^{plu} \equiv \frac{(1-\beta)}{2}$: $y_A^* = 0$ and $y_B^* = 0$;
- (ii) if $\beta > \beta^{plu} \equiv \frac{(1-\alpha)}{2}$: $y_A^* = -1$ and $y_B^* = 1$; and,
- (iii) if $\alpha > \alpha^{plu}$ and $\beta < \beta^{plu}$: $y_A^* = -1$ and $y_B^* = 0$.

Proposition 1 states that each candidate targets her base if and only if her base is sufficiently large relative to the share of centrists in the majority. To understand the result, we initially focus on advantaged candidate A , who wins the election if and only if two conditions hold.

First, A wins more first preferences than B if and only if (7) exceeds (8). That comparison yields a threshold $\tau_{plu}^{AB}(y_A, y_B)$ such that A defeats B if and only if the aggregate shock to moderates' preferences in favor of A is large enough: $\tau_0 \geq \tau_{plu}^{AB}$. Second, A wins more first preferences than C if and only if (7) exceeds C 's first preferences of $\gamma F(\theta)$. That comparison yields another preference shock threshold $\tau_{plu}^{AC}(y_A, y_B)$ such that A defeats C if and only if the aggregate shock in favor of A is large enough: $\tau_0 \geq \tau_{plu}^{AC}$.

So, under plurality rule, for any pair of platform $(y_A, y_B) \in \{-1, 0, 1\}^2$, A wins if and only if

$$\tau_0 > \max \left\{ \tau_{plu}^{AB}(y_A, y_B), \tau_{plu}^{AC}(y_A, y_B) \right\} = \tau_{plu}^{AC}(y_A, y_B). \quad (9)$$

Equality (9) reflects that A defeats B whenever she beats the front-runner candidate C . Moreover, $\gamma > \alpha$ implies that A cannot defeat front-runner C solely by turning out her own base: she requires the support of centrist group-zero voters. A maximizes her appeal to centrists by locating at $y_A = 0$: she lowers the threshold to win their first preferences, and in that

event she also raises centrists' enthusiasm and thus their turnout. If A instead targets her core supporters by locating at $y_A = -1$, she boosts their enthusiasm and thus their turnout.

Recall that A 's base is mass α and centrists are mass $1 - \alpha - \beta$. Under our assumption that $F(\rho)$ is uniform, A 's net gain in turnout from targeting core supporters versus centrists—conditional on both groups preferring her to B —is proportional to the difference in the share of supporters in each group: $\alpha - (1 - \alpha - \beta)$. When $\alpha < \alpha^{\text{plu}}$, this difference is negative and A focuses on mobilizing centrists at the expense of her core supporters. Otherwise, A targets her policy appeal to her base.

Similar considerations govern B 's platform choice: if her base is small relative to centrists, in the sense that $\beta - (1 - \alpha - \beta) < 0$, which is equivalent to $\beta < \beta^{\text{plu}}$, she focuses on mobilizing centrists. Otherwise, she targets her core supporters.

4. Equilibrium Under Ranked-Choice Voting

Under RCV, voters can express as many preferences as they wish. As under plurality rule, a group- j voter turns out if and only if her most-preferred candidate's payoff exceeds her reservation value. *Contrary to* plurality rule, however, voters may also express a preference for other candidates. A majority voter with reservation value ρ :

(1) abstains if she insufficiently enthusiastic about either candidate, which occurs if

$$\rho > \max\{u_j^A(y_A; \tau_j), u_j^B(y_B; \tau_j)\},$$

(2) casts a single preference vote only for her most-preferred candidate if she likes this candidate enough, but is insufficiently enthusiastic about the other majority candidate, which occurs if

$$\max\{u_j^A(y_A; \tau_j), u_j^B(y_B; \tau_j)\} > \rho > \min\{u_j^A(y_A; \tau_j), u_j^B(y_B; \tau_j)\},$$

(3) casts two preference votes (ranking the candidates in order of preference) if she is suf-

ficiently enthusiastic about both candidates, which occurs if

$$\rho < \min\{u_j^A(y_A; \tau_j), u_j^B(y_B; \tau_j)\}.$$

Notice that whether majority voters rank C third or simply do not rank her is immaterial in our setting.

A 's and B 's total first preferences under RCV are the same as under plurality, given by expressions (5) and (6). However, these candidates' total first *and* second preferences are:

$$\begin{aligned} v_A^s(y_A; \tau_0) &\equiv \sum_{j \in \{-1, 0, 1\}} \pi(j) F(u_j^A(y_A; \tau_j)) \\ &= \alpha F(\theta - 1 - y_A) + (1 - \alpha - \beta) F(\tau_0 - |y_A|) + \beta F(-\theta - 1 + y_A) \quad (10) \\ &> v_A^f(y_A, y_B; \tau_0). \end{aligned}$$

The key comparison with A 's first preferences given in expression (5) is that A wins second preferences even from voters that prefer B . Similarly, B 's total first and second preferences are

$$\begin{aligned} v_B^s(y_B; \tau_0) &= \sum_{j \in \{-1, 0, 1\}} \pi(j) F(u_j^B(y_B; \tau_j)) \\ &= \beta F(\theta - 1 + y_B) + (1 - \alpha - \beta) F(-\tau_0 - |y_B|) + \alpha F(-\theta - 1 - y_B). \quad (11) \end{aligned}$$

Recall that A 's base is size α , and B 's is size $\beta \leq \alpha$. A defeats B if and only if her first preferences exceed B 's, i.e., if and only if (7) exceeds (8). Further, A defeats C if and only if her total first and second preferences reflected in expression (10) exceed C 's: this implies that there exists a preference shock threshold $\tau_{rcv}^{AC}(y_A, y_B)$ such that A 's first and second preferences (10) exceed C 's if and only if $\tau_0 \geq \tau_{rcv}^{AC}$.

We conclude that A wins the election when the platforms are $(y_A, y_B) \in \{-1, 0, 1\}$ if and only if

$$\tau_0 > \max\{\tau_{plu}^{AB}(y_A, y_B), \tau_{rcv}^{AC}(y_A, y_B)\}. \quad (12)$$

Comparing (12) with the corresponding requirement under plurality rule in expression (9),

the crucial innovation under RCV is that if A defeats B , she wins the second preferences of any voter that preferred B to A , but liked A enough to cast a second preference. These second preferences are valuable in the contest against C .

By analogous reasoning, there exists a threshold $\tau_{\text{rcv}}^{BC}(y_A, y_B)$ such that B wins the election if and only if

$$\tau_0 < \min\{\tau_{\text{plu}}^{AB}(y_A, y_B), \tau_{\text{rcv}}^{BC}(y_A, y_B)\}. \quad (13)$$

An important preliminary observation is that a centrist electoral strategy always maximizes a candidate's first and second preferences—and thus her prospects of defeating C .

Observation 1. *Under RCV, a centrist policy $y_J = 0$ maximizes $J \in \{A, B\}$'s total first and second preference votes, regardless of her opponent's strategy.*

To understand why, recall that a voter awards a preference to a candidate if and only if that candidate's payoff exceeds the voter's reservation utility. Since reservation utilities are uniformly distributed, maximizing the sum of first and second preferences is equivalent to maximizing the sum of majority voters' payoffs. A centrist platform maximizes the sum of payoffs by minimizing the average distance between voters' preferred policies and the candidate's platform. This observation captures the intuition that RCV incentivizes moderation in the search for second preferences.

Nonetheless, this intuition is incomplete, because A only wins B 's second preferences if her *first* preferences exceed B 's. On the one hand, a centrist platform maximizes A 's chances of swaying centrists in the contest against B . On the other hand, failing to win over centrists in the contest against candidate B does not put centrists' support beyond A 's reach: A may secure enough of their second preferences—and even those of some of B 's core supporters—to defeat C . This lowers A 's electoral penalty from focusing on her core supporters in order to mobilize their turnout in the pairwise contest against B . So, Observation 1 does *not* imply that centrism maximizes A 's winning prospects.

Does the improved prospect of winning second preference votes encourage candidates A or B to moderate in pursuit of these second preferences? If weaker candidate B locates at

centrist platform $y_B = 0$, stronger A also prefers to adopt centrist platform $y_A = 0$ rather than focus on her base if and only if

$$\max \left\{ \tau_{\text{plu}}^{AB}(0, 0), \tau_{\text{rcv}}^{AC}(0, 0) \right\} \leq \max \left\{ \tau_{\text{plu}}^{AB}(-1, 0), \tau_{\text{rcv}}^{AC}(-1, 0) \right\}. \quad (14)$$

The LHS is A 's binding constraint to win with a centrist platform $y_A = 0$ —defeating centrist B with first preferences ($\tau_0 \geq \tau_{\text{plu}}^{AB}$) and defeating C with combined first and second preferences ($\tau_0 \geq \tau_{\text{rcv}}^{AC}$). The RHS denotes the corresponding constraint when A responds to B 's choice of $y_B = 0$ with a polarized platform $y_A = -1$.

Our next proposition establishes that this condition holds if and only if A 's base—and thus her advantage over B —is not too large.

Proposition 2. *If and only if candidate A 's base is **small**, in the sense that*

$$\alpha < \alpha^{\text{rcv}} \equiv \frac{(1 - \beta)F(0) + \beta F(\theta - 1)}{F(0) + F(\theta)},$$

then in the unique equilibrium under RCV both candidates target their policy appeal to centrists: $y_A^ = y_B^* = 0$.*

When $\alpha > \alpha^{\text{rcv}}$, A 's prospects of winning her pairwise contest against centrist B are greatest when A pursues a polarized platform. When A defeats B by way of a base strategy, she wins second preferences from both centrist voters and even some of B 's core supporters. In fact, the proposition's proof verifies that at the threshold shock realization $\tau_{\text{plu}}^{AB}(-1, 0)$ where A wins more first preferences than B , A 's total first and second preferences under RCV are also sufficient to defeat C .

Even though a base strategy sacrifices first preferences *and* some second preferences, the strategy can be electorally valuable in A 's effort to defeat B . Only if the imbalance favoring A 's base is small enough, A 's best path to defeat B is to compete with her directly for centrists' first preferences. This reflects that when imbalances in the size of each majority candidate's base are not too large, the resolution of moderate voters' first preference is decisive for their pairwise contest.

Changes in political primitives that reduce centrists' propensity to turn out in elections strengthen A 's advantage, and also her incentive to focus on her base.

Corollary 1. *Threshold α^{rcv} decreases whenever:*

1. *partisan attachments strengthen (θ increases), or*
2. *voter participation costs $\bar{\rho}$ increase.*

When centrists' relative propensity to turn out diminishes, the pairwise contest between A and B is increasingly decided by turnout from each candidate's core supporters. Stronger attachment (θ) amongst core supporters raises their relative turnout. Lower average voter participation or engagement reflected in higher $\bar{\rho}$ also lowers moderate voters' relative participation. Both changes intensify A 's advantage from a relatively larger base, and therefore allow A to defeat B with a base strategy even when the imbalance in her share of core supporters decreases.

In fact, if the imbalance between the majority candidates' core supporters is too large, advantaged A no longer prefers to compete directly with her weaker opponent B for centrists' preferences; instead, she abandons moderation and focuses exclusively on mobilizing her base.

Proposition 3. *There exists a threshold β^{rcv} such that if A 's base is **large** but B 's base is **small**, in the sense that*

$$\alpha > \alpha^{rcv} \text{ and } \beta < \beta^{rcv},$$

in the unique equilibrium under RCV, A targets her core supporters, while B targets centrists: $y_A^ = -1$ and $y_B^* = 0$.*

When A 's base is large enough, her strategy for defeating B reverts to mobilizing core supporters. Depending on the size of B 's base, weaker B may not face any trade-offs. Maintaining the uncontested centre-ground maximizes B 's total first and second preference votes, and thus her prospects of defeating C . And, when B 's base of size β is small enough, turning out centrists is also her best strategy for defeating A .

When B 's base is large, however, she faces a trade-off between a centrist strategy that maximizes her first and second preferences to defeat C , versus a base strategy that maximizes her prospects of defeating A with first preferences.

Proposition 4. *If both A 's and B 's bases are **large**: $\alpha \geq \alpha^{rcv}$ and $\beta \geq \beta^{rcv}$, the generically unique equilibrium under RCV is in mixed strategies: A randomizes over policies -1 and 0 , and B randomizes over policies 0 and 1 .*

To understand the necessity of randomization when the candidates' bases are large, recall that when $\alpha \geq \alpha^{rcv}$, the profile $(0, 0)$ is not an equilibrium because of A 's incentive to revert to her base, and when $\beta \geq \beta^{rcv}$, the profile $(-1, 0)$ is not an equilibrium because of B 's incentive to revert to her base. Why is a profile $(-1, 1)$ in which each candidate mobilizes her base not an equilibrium?⁵

If A and B both pursue a base strategy, A wins moderates' first preferences if and only if $\tau_0 \geq \hat{\tau}(0, 0) = 0$. If A instead reverts to moderates' preferred policy of zero, she alone appeals to moderates, and wins their first preferences whenever $\tau_0 \geq \hat{\tau}_0(-1, 0) = -1/2$. When τ_0 surpasses this threshold, A receives a discontinuous jump in her total first preferences—and since her base is larger than B 's, winning moderates' first preferences is sufficient for A to defeat B . This encourages A to revert to a centrist strategy. For the same reason, however, if B expects A to target moderates, B also prefers to target moderates. Thus, there cannot be a pure strategy equilibrium.

5. Policy Moderation Under Plurality versus RCV

We turn to the comparison of candidates' strategies under plurality and RCV. We first ask: does RCV better-incentivize candidates to jointly pursue moderate strategies instead of targeting their core supporters?

Proposition 1 highlights that centrist convergence is the unique equilibrium under plurality rule if and only if $\alpha < \alpha^{plu}$. Proposition 2 shows that centrist convergence is the unique

⁵The Appendix also rules out the remaining possibility for a pure strategy equilibrium in which $y_A = 0$ and $y_B = 1$.

equilibrium under RCV if and only if $\alpha < \alpha^{\text{rcv}}$. In both contexts, the binding constraint is A 's incentive to shift towards targeting her base instead of centrists. So, if threshold α^{rcv} exceeds α^{plu} , RCV provides the advantaged candidate A with stronger incentives than plurality to focus on moderates versus core supporters when she expects B to do the same.

Proposition 5. *RCV threshold α^{rcv} exceeds the corresponding plurality threshold α^{plu} if and only if B 's base is sufficiently large:*

$$\beta \geq \frac{F(\theta) - F(0)}{2F(\theta - 1) + F(\theta) - F(0)} \equiv \hat{\beta}(\theta, \bar{\rho}).$$

Threshold $\hat{\beta}$ decreases in voter engagement (lower $\bar{\rho}$) and increases in the strength of partisan attachments (θ).

Under plurality, A focuses on securing enough first preferences to defeat C . In that context, A defeats C *only if* she wins the support of moderate voters. She chooses policy to maximize overall turnout in the event that she wins first preferences from *both* her core supporters and moderate voters. The size of B 's base β plays no direct role in that calculation: only the allocation of the majority between A 's base α and centrists $1 - \alpha - \beta$ matters.

Under RCV, A focuses on securing enough first preferences to defeat B . When B 's base of size β is small, or when moderates are less prone to turn out, A 's ability to defeat B solely with the support of her own base increases, and so does her incentive to target her base. Thus, Proposition 5 highlights that A 's incentive to pursue a polarized platform is greater under RCV when β is low, in contexts where voter engagement is low ($\bar{\rho}$ large), and when the majority is segmented into partisan enclaves in which enthusiasm for one candidate over another is overwhelming (large θ).

We next turn to disadvantaged candidate B . Proposition 1 showed that when $\beta < \beta^{\text{plu}}$, under plurality rule B prefers to target moderates when A targets her core supporters. Otherwise, B prefers to target her own core supporters. Proposition 3 shows that under RCV, B prefers to target moderates when $\beta < \beta^{\text{rcv}}$. So, RCV disciplines disadvantaged candidate B to a greater extent than plurality rule if $\beta^{\text{rcv}} \geq \beta^{\text{plu}}$.

Proposition 6. RCV threshold β^{rcv} is *always* weakly larger than the corresponding plurality threshold β^{plu} .

To see why, recognize that B needs centrist voters' first preferences in order to defeat A . Platform $y_B = 1$ maximizes B 's core supporters turnout, while a platform $y_B = 0$ maximizes centrists' turnout. With uniform participation costs, the net turnout gain from a centrist strategy is positive if and only if their mass $1 - \alpha - \beta$ exceeds B 's core supporters, β . This is precisely the same condition required under plurality rule ($\beta^{plu} = (1 - \alpha)/2$). So, RCV cannot weaken B 's incentive to target moderates.

In fact, RCV may *strengthen* B 's incentive to target moderates. This is true whenever B 's binding constraint is not to defeat A with first preferences, but instead to defeat C with first and second preferences. Because the sum of B 's first and second preferences is always maximized with a centrist platform, weaker B may prefer to target centrists even when $\beta > \beta^{plu}$.

To summarize: RCV disciplines advantaged A to pursue moderation to a greater extent than plurality rule only when its majority opponent B 's base is not too small. Otherwise, RCV *intensifies* A 's incentive to target its core supporters. However, RCV *always* disciplines disadvantaged B to a greater extent than plurality rule. In fact, for any primitives, B 's strategy under plurality rule first-order stochastically dominates her strategy under RCV, but the same is not true for A .

6. Further Results

[Cormack \(2021\)](#) documents a raft of other important arguments proposed by RCV advocates. We use our results to assess two of these arguments.

Does RCV Increase Turnout? A prominent argument is that RCV increases voter participation. Yet existing studies report mixed findings—either no effect or even negative effects from the adoption of RCV ([McGinn 2020](#)). These equivocal findings can be rationalized in our framework.

Proposition 7. When $\beta > \beta^{plu}$, turnout is higher under plurality. When $\beta < \beta^{plu}$ and $\alpha^{rcv} < \alpha^{plu}$, there exists an open interval $[\alpha^\dagger, \alpha^{plu}]$ for α in which expected turnout is higher under RCV.

When both candidates' bases are large (i.e., if $\beta > \beta^{\text{plu}}$), turnout is maximized when each of A and B mobilizes core supporters. This is the equilibrium under plurality rule, in which the candidates maximize their first preference votes to defeat C .

Under RCV, there is never an equilibrium in which both candidates exclusively mobilize core supporters. When $\beta^{\text{plu}} < \beta < \beta^{\text{rcv}}$, B targets centrists in order to bolster her *second* preferences in the event she defeats A . She wins these second preferences only from voters that *already* turn out, and therefore wins them at the expense of increasing first preferences—and therefore turnout—from her base. Finally, when $\beta > \beta^{\text{rcv}}$ the candidates randomize between mobilization strategies. Each candidate does so to ensure that her opponent doesn't win moderates' first preferences: even when moderates are the smallest majority group, they play an outsized role in the pairwise contest between A and B .

Nonetheless, the proposition unearths circumstances in which RCV raises expected turnout. This arises for some $\alpha \in [\alpha^{\text{rcv}}, \alpha^{\text{plu}}]$ —i.e., it arises for primitives in which both plurality induces both candidates to choose moderate platforms, but RCV leads A to focus on her base.

To understand why, recognize that the plurality threshold α^{plu} is determined by the condition that A defeats C when she mobilizes her core supporters *and* receives some support from moderates. By contrast, the RCV threshold α^{rcv} is determined by the condition that A can defeat B when she mobilizes her core supporters, without any support from moderates. Proposition 5 shows $\alpha^{\text{rcv}} < \alpha^{\text{plu}}$ in contexts with strong partisan attachments or low average voter engagement. In these contexts, core supporters are easier to mobilize. So, the marginal increase in turnout from a base strategy overwhelms the corresponding increase from moderation—even if the candidates' core supporters are a minority.

When Does RCV Fail to Produce a Majority Winner? With three or more candidates, the winner under plurality may fail to win a majority—the so-called “spoiler effect.” In making its case for RCV, the New York Times Editorial Board asserted that “Ranked-choice voting solves this problem” (New York Times 2019). We show it may not, and especially in contexts with strong partisan attachments.

Proposition 8. *There exists γ^* , decreasing in polarization θ , such that if and only if $\gamma > \gamma^*$, under RCV there is a positive probability that fewer than a majority of voters that turn out cast a vote for the winning candidate.*

When the majority is segmented into enclaves of candidates' core supporters, voters that show up to vote for their most-preferred candidate are increasingly hostile to other candidates. This increases the prospect that they do not fully utilize the ballot, and thus that their votes are "wasted" after their first-ranked candidate is eliminated.

7. Conclusion

Our paper studies electoral competition under Ranked Choice Voting (RCV). We ask: When does RCV provide greater incentives for candidates to moderate their policy platforms than plurality rule? Does RCV necessarily increase voter participation? And, under what conditions does RCV lead to the election of a candidate with the support of a majority of voters?

RCV provides greater incentives to candidates to moderate their platforms when the imbalances in the shares of core supporters are not too large. Indeed, candidates with a relatively small core vote are more inclined to moderate their platforms. But, if this imbalance is large enough, the candidate with a larger share of core supporters has a greater incentive to pursue extreme policies in order to mobilize that base. This tendency increases when the candidates' core supporters display stronger partisan attachments, or in contexts of low voter engagement—for example, because of low information or limited interest in the contest. Stronger partisan attachments also increase the prospect that the winning candidate fails to command a majority of support amongst all votes cast. Notably, these are precisely the contexts in which RCV proponents argue that the reform is most urgently needed. Finally, RCV may increase *or* decrease voter turnout relative to plurality rule.

We close with a broader interpretation of our results, and how they relate to existing arguments that favor RCV's adoption. By allowing voters to express a preference for multiple candidates, RCV implicitly helps voters to solve a coordination problem they would otherwise face in multi-candidate elections under plurality rule. For a fixed set of alternatives, this

improved implicit coordination facilitates the election of moderate policies, and in particular majority-preferred policies when they exist. However, this improved implicit coordination also changes the candidates' strategies, by opening up new pathways to electoral victory that may be absent under plurality. Changes in electoral rules therefore have the potential to create new conflicts between candidates whose consequences can be difficult to predict. Indeed, those consequences may be opposite to the aspirations of both scholars and reformers of electoral systems.

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Appendix: Proofs of Propositions

Parameter Restrictions. We remind the reader of the following assumptions:

- A1. *A's base exceeds B's:* without loss of generality, $\alpha \geq \beta$
- A2. *Bases are loyal:* $\theta > 2$, ensures that A and B 's bases are loyal;
- A3. *Interior turnout:* $\min \left\{ \frac{1}{2\phi} - \bar{\rho}, \bar{\rho} + \frac{1}{2\phi} \right\} > \theta + 2$;
- A4. *Interior winning probabilities:* $(1 - \alpha - \gamma)F(0) > \phi$.
- A5. *Divided majority:* $(1 - \alpha - \gamma)F(0) < \phi \left[\theta(\gamma - \alpha) + \min \left\{ \alpha, \frac{1 - \alpha - \beta}{2} \right\} \right] - (\alpha - \beta)F(0)$.

Note that A5 further implies that C 's base is largest: $\gamma > \alpha$. We assume without loss of generality that ties between A and B are resolved in favor of A and ties between C and either A and B are resolved against C .

We interpret the mass γ of C -voters as her base and implicitly assume C always locates at their bliss point. A2 implies that $y_A = 1$ and $y_B = -1$ are strictly dominated. As a result, $y_A \in \{-1, 0\}$, $y_B \in \{0, 1\}$ and the share of first-preference votes of each candidate can be written as:

$$\begin{aligned} v_A^f &= \alpha F(\theta - 1 - y_A) + \mathbb{I} \left\{ \tau \geq \frac{-y_A - y_B}{2} \right\} (1 - \alpha - \beta) F(\tau + y_A) \\ v_B^f &= \beta F(\theta - 1 + y_B) + \mathbb{I} \left\{ \tau < \frac{-y_A - y_B}{2} \right\} (1 - \alpha - \beta) F(-\tau - y_B) \\ v_C^f &= \gamma F(\theta). \end{aligned}$$

Similarly, the share of second-preference votes of each candidate can be written as:

$$\begin{aligned} v_A^s &= \alpha F(\theta - 1 - y_A) + (1 - \alpha - \beta) F(\tau + y_A) + \beta F(-\theta - 1 + y_A) \\ v_B^s &= \beta F(\theta - 1 + y_B) + (1 - \alpha - \beta) F(-\tau - y_B) + \alpha F(-\theta - 1 - y_B) \\ v_C^s &= v_C^f = \gamma F(\theta). \end{aligned}$$

Proof of Proposition 1. We first show that, under our assumptions, each candidate A or B wins under plurality if and only if she wins more votes than C .

Lemma 1. For any $(y_A, y_B) \in \{-1, 0\} \times \{0, 1\}$ and $J \in \{A, B\}$, $v_J^f(y_A, y_B) \geq v_C^f$ only if $v_J^f(y_A, y_B) \geq v_{-J}^f(y_A, y_B)$, where $-J = \{A, B\} \setminus \{J\}$.

Proof. A5 implies $\gamma > \alpha$, so $v_A^f \geq v_C^f$ requires $\tau \geq \frac{-y_A - y_B}{2}$, which implies that $v_B^f = \beta F(\theta - 1 + y_B) < v_C^f$, the last inequality following from combining A1 and $\alpha < \gamma$. Using the same argument, $v_B^f \geq v_C^f$ requires $\tau < \frac{-y_A - y_B}{2}$, which implies $v_A^f = \alpha F(\theta - 1 - y_A) < v_C^f$. \square

Lemma 1 implies that we can write $\pi_A^{plu} = \Pr(v_A^f \geq v_C^f) = \Pr(\tau \geq \hat{\tau}_{y_A}^A)$ and $\pi_B^{plu} = \Pr(v_B^f \geq v_C^f) = \Pr(\tau < \hat{\tau}_{y_B}^B)$, where the thresholds solve

$$\begin{aligned} (1 - \alpha - \beta)F(\hat{\tau}_{-1}^A) &\equiv Z(\hat{\tau}_{-1}^A) = \gamma F(\theta) - \alpha F(\theta) + \phi(1 - \alpha - \beta) \\ Z(\hat{\tau}_0^A) &= \gamma F(\theta) - \alpha F(\theta) + \phi\alpha \\ Z(-\hat{\tau}_1^B) &= \gamma F(\theta) - \beta F(\theta) + \phi(1 - \alpha - \beta) \\ Z(-\hat{\tau}_0^B) &= \gamma F(\theta) - \beta F(\theta) + \phi\beta \end{aligned}$$

By inspection, it's easy to see that $-\hat{\tau}_1^B > \hat{\tau}_{-1}^A$ and $-\hat{\tau}_0^B > \hat{\tau}_0^A$. Moreover, A4 implies that $\min\{\hat{\tau}_0^B, \hat{\tau}_1^B\} > -\theta$, since

$$\begin{aligned} \max\{Z(-\hat{\tau}_0^B), Z(-\hat{\tau}_1^B)\} &= (\gamma - \beta)F(\theta) + \phi \max\{\beta, 1 - \alpha - \beta\} \\ &< (\gamma - \beta)F(\theta) + \phi < (\gamma - \beta)F(\theta) + (1 - \alpha - \gamma)F(0) < (1 - \alpha - \beta)F(\theta) = Z(\theta) \end{aligned}$$

Finally, A5 implies that $\hat{\tau}_0^A > 0$ (which implies $\hat{\tau}_0^B < 0$) and $\hat{\tau}_{-1}^A > \frac{1}{2}$ (which implies $\hat{\tau}_1^B < -\frac{1}{2}$).

To see this, notice that A5 can be rewritten as follows:

$$\begin{aligned} (1 - \alpha - \gamma)F(0) &< \phi \left[\theta(\gamma - \alpha) + \min \left\{ \alpha, \frac{1 - \alpha - \beta}{2} \right\} \right] - (\alpha - \beta)F(0) \\ \Leftrightarrow (1 - \alpha)F(0) + \phi\theta\alpha - \phi \min \left\{ \alpha, \frac{1 - \alpha - \beta}{2} \right\} + (\alpha - \beta)F(0) &< \gamma F(\theta) \\ \Leftrightarrow (1 - \alpha - \beta)F(0) + \alpha F(\theta) - \phi \min \left\{ \alpha, \frac{1 - \alpha - \beta}{2} \right\} &< \gamma F(\theta) \end{aligned}$$

$$\Leftrightarrow \max \{(1 - \alpha - \beta)F(0) + \alpha F(\theta - 1), (1 - \alpha - \beta)F(-1/2) + \alpha F(\theta)\} < \gamma F(\theta)$$

which implies that $Z(0) < \gamma F(\theta) - \alpha F(\theta - 1)$ and $Z(\frac{1}{2}) < \gamma F(\theta) - \alpha F(\theta)$.

As a result, $y_A = 0$ is individually rational if and only if $\hat{\tau}_{-1}^A \geq \hat{\tau}_0^A$ and $y_B = 0$ is individually rational if and only if $\hat{\tau}_1^B \leq \hat{\tau}_0^B$. This observation directly implies the proposition. \square

Proof of Propositions 2, 3 and 4.

Pivotal events. We begin by studying each candidate's electoral chances under each of the four possible pairs (y_A, y_B) .

Centrist profile. If $y_A = y_B = 0$, A defeats B if and only if $\tau \geq \min\{0, \tau_{0,0}^{AB}\}$, where $\tau_{0,0}^{AB}$ solves $v_A^f(0, 0) = v_B^f(0, 0)|_{\tau < 0}$, i.e.,

$$Z(-\tau_{0,0}^{AB}) = \alpha F(\theta - 1) - \beta F(\theta - 1).$$

Further, A defeats C if and only if $\tau \geq \tau_0^A$, where τ_0^A solves $v_A^s(0, 0) = v_C^f$, i.e.,

$$Z(\tau_0^A) = \gamma F(\theta) - \alpha F(\theta - 1) - \beta F(-\theta - 1).$$

Finally, B defeats A if and only if $\tau < \min\{0, \tau_{0,0}^{AB}\}$ and defeats C if and only if $\tau \leq \tau_0^B$, where τ_0^B solves $v_B^s(0, 0) = v_C^f$, i.e.,

$$Z(-\tau_0^B) = \gamma F(\theta) - \alpha F(-\theta - 1) - \beta F(\theta - 1).$$

So, when $y_A = y_B = 0$, A wins the election if and only if $\tau \geq \max\{\min\{0, \tau_{0,0}^{AB}\}, \tau_0^A\}$, and B wins the election if and only if $\tau \leq \min\{0, \tau_{0,0}^{AB}, \tau_0^B\}$.

Asymmetric profile, A base. If $y_A = -1$ and $y_B = 0$, A defeats B if and only if $\tau \geq \min\{\frac{1}{2}, \tau_{-1,0}^{AB}\}$, where $\tau_{-1,0}^{AB}$ solves $v_A^f(-1, 0) = v_B^f(-1, 0)|_{\tau < \frac{1}{2}}$, i.e.,

$$Z(-\tau_{-1,0}^{AB}) = \alpha F(\theta) - \beta F(\theta - 1).$$

Further, A defeats C if and only if $\tau \geq \tau_{-1}^A$, where τ_{-1}^A solves $v_A^s(-1, 0) = v_C^f$, i.e.,

$$Z(\tau_{-1}^A) = \gamma F(\theta) - \alpha F(\theta) - \beta F(-\theta - 2) + \phi(1 - \alpha - \beta).$$

Finally, B defeats A if and only if $\tau < \min\{\frac{1}{2}, \tau_{-1,0}^{AB}\}$ and defeats C if and only if $\tau \leq \tau_0^B$. So, when $y_A = -1$ and $y_B = 0$, A wins the election if and only if $\tau \geq \max\{\min\{\frac{1}{2}, \tau_{-1,0}^{AB}\}, \tau_{-1}^A\}$, and B wins the election if and only if $\tau \leq \min\{\frac{1}{2}, \tau_{-1,0}^{AB}, \tau_0^B\}$.

Base profile. If $y_A = -1$ and $y_B = 1$, A defeats B if and only if $\tau \geq \min\{0, \tau_{-1,1}^{AB}\}$, where $\tau_{-1,1}^{AB}$ solves $v_A^f(-1, 1) = v_B^f(-1, 1)|_{\tau < 0}$, i.e.,

$$Z(-\tau_{-1,1}^{AB}) = \alpha F(\theta) - \beta F(\theta - 1) + \phi(1 - \alpha - \beta).$$

Further, A defeats C if and only if $\tau \geq \tau_{-1}^A$. Finally, B defeats A if and only if $\tau < \min\{0, \tau_{-1,1}^{AB}\}$ and defeats C if and only if $\tau \leq \tau_1^B$, where τ_1^B solves $v_B^s(-1, 1) = v_C^f$, i.e.,

$$Z(-\tau_1^B) = \gamma F(\theta) - \alpha F(-\theta - 2) - \beta F(\theta) + \phi(1 - \alpha - \beta).$$

So, when $y_A = -1$ and $y_B = 1$, A wins the election if and only if $\tau \geq \max\{\min\{0, \tau_{-1,1}^{AB}\}, \tau_{-1}^A\}$, and B wins the election if and only if $\tau \leq \min\{0, \tau_{-1,1}^{AB}, \tau_1^B\}$.

Asymmetric profile, B base. If $y_A = 0$ and $y_B = 1$, A defeats B if and only if $\tau > \tau_{0,1}^{AB}$, where

$$\tau_{0,1}^{AB} \equiv \begin{cases} \hat{\tau}_{0,1}^{AB} & \text{if } \beta F(\theta) > \alpha F(\theta - 1) + (1 - \alpha - \beta)F(-\frac{1}{2}) \\ -\frac{1}{2} & \text{if } \beta F(\theta) - (1 - \alpha - \beta)F(-\frac{1}{2}) < \alpha F(\theta - 1) < \beta F(\theta) + (1 - \alpha - \beta)F(-\frac{1}{2}) \\ \check{\tau}_{0,1}^{AB} & \text{if } \beta F(\theta) + (1 - \alpha - \beta)F(-\frac{1}{2}) < \alpha F(\theta - 1), \end{cases}$$

where $\hat{\tau}_{0,1}^{BA} > -\frac{1}{2}$ solves

$$Z(\hat{\tau}_{0,1}^{BA}) = \beta F(\theta) - \alpha F(\theta - 1)$$

and $\check{\tau}_{0,1}^{BA} < -\frac{1}{2}$ solves

$$Z(-\check{\tau}_{0,1}^{BA}) = \alpha F(\theta - 1) - \beta F(\theta) + \phi(1 - \alpha - \beta)$$

$\beta F(\theta) + (1 - \alpha - \beta)F(-\tau - 1) - \alpha F(\theta - 1) = 0$. Thus, A wins the election if and only if $\tau > \max\{\tau_{0,1}^{AB}, \tau_0^A\}$, and B wins the election if and only if $\tau < \min\{\tau_{0,1}^{AB}, \tau_1^B\}$.

Observations. Before proceeding, we list a series of observations on how the thresholds derived above compare. First, notice that for any $x \in [-\theta - 2, \theta + 2]$,

$$F(-x) = \frac{1}{2} + \phi(-x - \bar{\rho}) = 1 - \left[\frac{1}{2} + \phi(x - \bar{\rho}) \right] - 2\phi\bar{\rho} = 2F(0) - F(x).$$

As a consequence, we have

$$Z(-x) = (1 - \alpha - \beta)2F(0) - Z(x).$$

Observation 2. $-\tau_0^B > \tau_0^A$

Follows from $Z(-\tau_0^B) - Z(\tau_0^A) = (\alpha - \beta)(F(\theta - 1) - F(-\theta - 1)) = \phi(\alpha - \beta)2\theta > 0$.

Observation 3. $\tau_{0,0}^{AB} > \tau_{-1,0}^{AB}$

Follows from $Z(-\tau_{-1,0}^{AB}) - Z(-\tau_{0,0}^{AB}) = \alpha(F(\theta) - F(\theta - 1)) = \phi\alpha > 0$.

Observation 4. $\tau_{-1}^A > \tau_0^A$

Follows from $Z(\tau_{-1}^A) - Z(\tau_0^A) = -\phi\alpha - \phi\beta + \phi(1 - \alpha - \beta) = \phi(1 - 2\alpha) > 0$, where the last inequality follows from the fact that A4 and A5's implication that $\gamma > \alpha$ imply $1 - 2\alpha > 1 - \alpha - \gamma > 0$.

Observation 5. $\tau_{0,0}^{AB} > \tau_{-1,1}^{AB}$

Follows from $Z(-\tau_{-1,1}^{AB}) - Z(-\tau_{0,0}^{AB}) = \phi(1 - \alpha - \beta) + \phi\alpha - \phi\beta = \phi(1 - 2\beta) > 0$, which follows from A1 and the fact that $\frac{1}{2} > \alpha$, established in the proof of Observation 5.

Observation 6. $\tau_0^B > \tau_1^B$

Follows from $Z(-\tau_1^B) - Z(-\tau_0^B) = \phi(1 - \alpha - \beta) + \phi\alpha + \phi\beta = \phi(1 - 2\beta) > 0$.

Observation 7. $\min\{0, \tau_{-1,1}^{AB}\} > \tau_{0,1}^{AB}$

Follows from the fact that we have $Z(\tau_{0,1}^{AB}) \leq \beta F(\theta) - \alpha F(\theta) + \phi\alpha$ and we can write $Z(\tau_{0,1}^{AB}) = (1 - \alpha - \beta)2F(0) - \alpha F(\theta - 1) + \beta F(\theta - 1)$. Hence:

$$Z(\tau_{-1,1}^{AB}) - Z(\tau_{0,1}^{AB}) \geq (1 - \alpha - \beta)(2F(0) - \phi) - \phi\alpha = (1 - \alpha - \beta)2F(0) - (1 - \beta)\phi > \phi > 0$$

$$Z(0) - Z(\tau_{0,1}^{AB}) \geq (1 - \alpha - \beta)F(0) - \beta F(\theta) + \alpha F(\theta) - \phi\alpha > (\alpha - \beta)F(\theta) + \phi(1 - \alpha) > 0,$$

where both inequalities follow from the fact that by A1, A4 and A5: $(1 - \alpha - \beta)F(0) > (1 - \alpha - \gamma)F(0) > \phi$.

Observation 8. $\tau_{-1,0}^{AB} > \tau_{-1}^A$.

Follows from

$$\begin{aligned} Z(\tau_{-1,0}^{AB}) - Z(\tau_{-1}^A) &= (2F(0) - \phi)(1 - \alpha - \beta) + \beta[F(\theta - 1) + F(-\theta - 2)] - \gamma F(\theta) \\ &= (1 - \alpha - \beta)2F(0) - (1 - \alpha)\phi + \beta[F(\theta) + F(-\theta - 2)] - \gamma F(\theta) \\ &= (1 - \alpha - \beta)2F(0) - (1 - \alpha)\phi + \beta[2F(0) - 2\phi] - \gamma F(\theta) \\ &= ((1 - \alpha) + 1 - \alpha - \gamma)F(0) - (1 - \alpha)\phi - \beta 2\phi - \gamma\phi\theta \\ &> \phi + (1 - \alpha)[F(0) - \phi] - \beta 2\phi - \gamma\phi\theta \\ &= \phi + (1 - \alpha)[F(-\theta - 2) + \phi(\theta + 1)] - \beta 2\phi - \gamma\phi\theta \\ &> \phi \left(1 + (1 - \alpha)(\theta + 1) - 2\beta - \gamma\theta \right) = \phi \left((1 - \alpha - \gamma)\theta + 2 - \alpha - 2\beta \right) > 0 \end{aligned}$$

where the first inequality follows from A4, the second from A3, and the last from A4 and $\frac{1}{2} > \alpha \geq \beta$.

Observation 9. $-\tau_1^B > \tau_{-1}^A$

Follows from $Z(-\tau_1^B) - Z(\tau_{-1}^A) = \alpha F(\theta) + \beta F(-\theta - 2) - \alpha F(-\theta - 2) - \beta F(\theta) = 2\phi(\alpha - \beta)$.

Centrist Equilibrium. By the previous steps, this equilibrium exists if and only if

$$\max\{\min\{0, \tau_{0,0}^{AB}\}, \tau_0^A\} \leq \max\{\min\{\tau_{-1,0}^{AB}, \frac{1}{2}\}, \tau_{-1}^A\}, \quad (15)$$

$$\min\{0, \tau_{0,0}^{AB}, \tau_0^B\} \geq \min\{\tau_{0,1}^{AB}, \tau_1^B\}, \quad (16)$$

where the first condition necessary and sufficient for $y_A = 0$ to be a best response to $y_B = 0$, and the second condition is necessary and sufficient for $y_B = 0$ to be a best response to $y_A = 0$.

Lemma 2. *Inequality (15) holds if and only if*

$$\max\{\tau_{-1,0}^{AB}, \tau_{-1}^A\} \geq \min\{0, \tau_{0,0}^{AB}\}. \quad (17)$$

Proof. Sufficiency. By Observation 4 $\tau_{-1}^A > \tau_0^A$, so if $\tau_{-1}^A \geq \min\{0, \tau_{0,0}^{AB}\}$, then (15) holds. If instead $\tau_{-1}^A < \min\{0, \tau_{0,0}^{AB}\}$, then using Observation 4, the fact that $\min\{0, \tau_{0,0}^{AB}\} < \frac{1}{2}$ and the fact that for any triplet $\{x, y, z\}$ we have $\max\{x, \min\{y, z\}\} = \min\{\max\{x, y\}, \min\{x, z\}\} \geq \min\{y, \min\{x, z\}\}$, we have

$$LHS(15) \geq \min\left\{\frac{1}{2}, \max\{\tau_{-1}^A, \tau_{-1,0}^{AB}\}\right\} \geq \min\{0, \tau_{0,0}^{AB}\} = RHS(15)$$

Necessity. Suppose that $\max\{\tau_{-1}^A, \tau_{-1,0}^{AB}\} < \min\{0, \tau_{0,0}^{AB}\}$ then $\tau_{-1,0}^{AB} < \frac{1}{2}$ and thus

$$LHS(15) = \max\{\tau_{-1}^A, \tau_{-1,0}^{AB}\} < \min\{0, \tau_{0,0}^{AB}\} \leq RHS(15)$$

which implies that (15) does not hold. □

Lemma 3. $\max\{\tau_{-1,0}^{AB}, \tau_{-1}^A\} = \tau_{-1,0}^{AB}$.

Proof. Follows directly from Observation 8. □

Lemma 4. *Inequality (16) holds if and only if*

$$\min\{\tau_{0,1}^{AB}, \tau_1^B\} \leq \min\{0, \tau_{0,0}^{AB}\} \tag{18}$$

Proof. Sufficiency. By Observation 6 $\tau_0^B > \tau_1^B \geq \min\{\tau_{0,1}^{AB}, \tau_1^B\}$. If in addition (18) holds, then we obtain (16).

Necessity. Suppose that $\min\{\tau_{0,1}^{AB}, \tau_1^B\} > \min\{0, \tau_{0,0}^{AB}\}$. Then $\tau_0^B > \tau_1^B \geq \min\{\tau_{0,1}^{AB}, \tau_1^B\}$ implies

$$LHS(16) = \min\{0, \tau_{0,0}^{AB}\} < \min\{\tau_{0,1}^{AB}, \tau_1^B\} = RHS(16)$$

which implies that (16) does not hold. □

Lemma 5. $\tau_{-1,0}^{AB} \geq \min\{0, \tau_{0,0}^{AB}\}$ implies $\min\{\tau_{0,1}^{AB}, \tau_1^B\} \leq \min\{0, \tau_{0,0}^{AB}\}$.

Proof. By Observation 2, $\tau_{-1,0}^{AB} < \tau_{0,0}^{AB}$, and thus we must have $\min\{0, \tau_{0,0}^{AB}\} = 0$. Then $\tau_{-1,0}^{AB} \geq 0$

implies $\tau_{0,1}^{AB} \leq 0$. To see that, notice that, after rearranging

$$Z(-\tau_{-1,0}^{AB}) = \alpha F(\theta) - \beta F(\theta - 1) \geq \beta F(\theta) - \alpha F(\theta - 1) \leq Z(\tau_{0,1}^{AB})$$

Hence, $Z(-\tau_{-1,0}^{AB}) \leq Z(0) \Leftrightarrow Z(\tau_{0,1}^{AB}) \leq Z(0)$, which implies $\tau_{0,1}^{AB} \leq 0$. \square

Corollary 2. *There exists $\alpha^{rcv} \in (0, \frac{1}{2})$ such that a centrist equilibrium exists if and only if $\alpha \leq \alpha^{rcv}$.*

Proof. Using the previous Lemmas, a centrist equilibrium exists if and only if $\tau_{-1,0}^{AB} \geq \min\{0, \tau_{0,0}^{AB}\}$, which can hold if and only if $\tau_{-1,0}^{AB} \geq 0$. Rearranging the corresponding indifference condition $Z(-\tau_{-1,0}^{AB}) = Z(0)$, we obtain:

$$\begin{aligned} \alpha F(\theta) - \beta F(\theta - 1) &= (1 - \alpha - \beta)F(0) \\ \Leftrightarrow \alpha^{rcv} &= \frac{(1 - \beta)F(0) + \beta F(\theta - 1)}{F(\theta) + F(0)} = \frac{F(0) + \beta\phi(\theta - 1)}{2F(0) + \phi\theta} < \frac{F(0) + \beta\phi(\theta - 1)}{2F(0) + 2\beta\phi\theta} < \frac{1}{2} \end{aligned}$$

\square

Asymmetric Equilibria. We now show that $(y_A, y_B) = (0, 1)$ cannot be an equilibrium and provide necessary and sufficient conditions for the existence of an equilibrium with $(y_A, y_B) = (-1, 0)$.

Lemma 6. *A strategy profile $y_A = 0$ and $y_B = 1$ is not an equilibrium.*

Proof. By the previous steps, this equilibrium requires the weak inequality (18 (which holds if and only if 16 holds) to be reversed, that is

$$\min\{\tau_{0,1}^{AB}, \tau_1^B\} \geq \min\{0, \tau_{0,0}^{AB}\}.$$

This, however, contradicts Observation 8, which shows that $\tau_{0,1}^{AB} < \min\{0, \tau_{0,0}^{AB}\}$. \square

We then consider an asymmetric equilibrium with $y_A = -1$ and $y_B = 0$. By the previous steps, this equilibrium exists if and only if

$$\tau_{-1,0}^{AB} \leq 0, \tag{19}$$

$$\min \left\{ \frac{1}{2}, \tau_{-1,0}^{AB}, \tau_0^B \right\} \geq \min \{0, \tau_{-1,1}^{AB}, \tau_1^B\}, \quad (20)$$

where the first condition necessary and sufficient for $y_A = -1$ to be a best response to $y_B = 0$, and the second condition is necessary and sufficient for $y_B = 0$ to be a best response to $y_A = -1$.

Lemma 7. *If Inequality (19) holds, Inequality (20) holds if and only if $\tau_{-1,0}^{AB} \geq \min\{0, \tau_{-1,1}^{AB}, \tau_1^B\}$.*

Proof. By Inequality (19), $\tau_{-1,0}^{AB} \leq 0 < \frac{1}{2}$ and thus Inequality (20) holds if and only if

$$\min\{\tau_{-1,0}^{AB}, \tau_0^B\} \geq \min\{0, \tau_{-1,1}^{AB}, \tau_1^B\} \quad (21)$$

Sufficiency. By Observation 6, $\tau_0^B > \tau_1^B$, which implies that $\tau_0^B \geq \min\{0, \tau_1^B\}$. If, in addition, $\tau_{-1,0}^{AB} \geq \min\{0, \tau_{-1,1}^{AB}, \tau_1^B\}$, then we obtain Inequality (21).

Necessity. Suppose that $\tau_{-1,0}^{AB} < \min\{0, \tau_{-1,1}^{AB}, \tau_1^B\}$. Then we have

$$LHS(21) \leq \tau_{-1,0}^{AB} < RHS(21).$$

Notice that we also need $\tau_{-1,0}^{AB} \leq 0$. So unless $\tau_{-1,0}^{AB} = 0 \leq \min\{\tau_{-1,1}^{AB}, \tau_1^B\}$ (a zero-measure set of the parameter space),⁶ Inequality (20) fails whenever $\tau_{-1,0}^{AB} < \min\{0, \tau_{-1,1}^{AB}, \tau_1^B\}$. \square

Corollary 3. *There exists $\beta^{rcv} \in [\frac{1-\alpha}{2}, \alpha]$ such that an asymmetric equilibrium with $y_A = -1$ and $y_B = 0$ exists if and only if $\alpha \geq \alpha^{rcv}$ and $\beta \leq \beta^{rcv}$.*

Proof. Using the previous Lemmas, we know that the asymmetric equilibrium exists if and only if $\min\{\tau_{-1,1}^{AB}, \tau_1^B, 0\} \leq \tau_{-1,0}^{AB} \leq 0$. The second inequality is guaranteed by $\alpha \geq \alpha^{rcv}$. The first inequality is equivalent to $\max\{Z(0), Z(-\tau_{-1,1}^{AB}), Z(-\tau_1^B)\} \geq Z(\tau_{-1,0}^{AB})$, that is:

$$\beta \leq \max \left\{ \frac{(1-2\alpha)F(0)}{\phi} - \phi(\alpha - \beta)\theta, 1 - \alpha - \beta, \frac{(\gamma - 2\alpha)F(0)}{\phi} + 1 + \alpha - \beta + \gamma\theta \right\} \Leftrightarrow \beta \leq \beta^{rcv}$$

Notice that, by inspection, $\beta^{rcv} \geq \frac{1-\alpha}{2}$. \square

⁶ Notice that $0 \leq \min\{\tau_{-1,1}^{AB}, \tau_1^B\}$ also implies $\tau_0^B > 0$ since $\tau_0^B > \tau_1^B$.

Other Equilibria. We now complete the equilibrium characterization by showing that there is no base equilibrium.

Lemma 8. *A strategy profile $y_A = -1$ and $y_B = 1$ is not an equilibrium.*

Proof. $y_A = -1$ is a best response to $y_B = +1$ if and only if

$$\max\{\min\{\tau_{-1,1}^{AB}, 0\}, \tau_{-1}^A\} \leq \max\{\tau_{0,1}^{AB}, \tau_0^A\}. \quad (22)$$

By Observation 4 $\tau_0^A < \tau_{-1}^A$ and by Observation 7 $\tau_{0,1}^{AB} < \min\{\tau_{-1,1}^{AB}, 0\}$. Hence, Inequality (22) fails. \square

Lemma 9. *When $\alpha > \alpha^{\text{rcv}}$ and $\beta > \beta^{\text{rcv}}$, there is a unique mixed strategy equilibrium in which A randomizes over $\{-1, 0\}$ and B randomizes over $\{0, 1\}$.*

Proof. Recall that $\pi_J(y_A, y_B)$ denote $J \in \{A, B\}$'s probability of winning when the platforms are (y_A, y_B) . Notice that:

$$\alpha > \alpha^{\text{rcv}} \iff \pi_A(-1, 0) > \pi_A(0, 0)$$

$$\text{Lemma 8} \iff \pi_A(-1, 1) < \pi_A(0, 1)$$

$$\beta > \beta^{\text{rcv}} \iff \pi_B(-1, 1) > \pi_B(-1, 0)$$

$$\text{Lemma 6} \iff \pi_B(0, 1) < \pi_B(0, 0)$$

Hence, there is no pure strategy equilibrium. Observe that A is indifferent between platforms in $\{-1, 0\}$ if and only if there exists $\sigma_B \in [0, 1]$ such that

$$\begin{aligned} \sigma_B \pi_A(-1, 0) + (1 - \sigma_B) \pi_A(-1, 1) &= \sigma_B \pi_A(0, 0) + (1 - \sigma_B) \pi_A(0, 1) \\ \iff \sigma_B &= \frac{\pi_A(0, 1) - \pi_A(-1, 1)}{\pi_A(0, 1) - \pi_A(-1, 1) + \pi_A(-1, 0) - \pi_A(0, 0)}. \end{aligned}$$

B is indifferent between platforms in $\{0, 1\}$ if and only if there exists $\sigma_A \in [0, 1]$ such that

$$\sigma_A \pi_B(0, 1) + (1 - \sigma_A) \pi_B(-1, 1) = \sigma_A \pi_B(0, 0) + (1 - \sigma_A) \pi_B(-1, 0)$$

$$\Leftrightarrow \sigma_A = \frac{\pi_B(-1, 1) - \pi_B(-1, 0)}{\pi_B(-1, 1) - \pi_B(-1, 0) + \pi_B(0, 0) - \pi_B(0, 1)}.$$

Under the assumptions, there exists a unique pair of $(\sigma_A, \sigma_B) \in [0, 1]^2$ that induces the above-described indifference. \square

Lemma 10. *When either (i) $\alpha < \alpha^{\text{rcv}}$ or (ii) $\alpha > \alpha^{\text{rcv}}$ and $\beta < \beta^{\text{rcv}}$, there is no equilibrium in which either candidate randomizes.*

Proof. Notice that when $\alpha < \alpha^{\text{rcv}}$, Lemma 8 implies that $y_A = -1$ is a strictly dominant strategy for A . Moreover, Lemma 6 implies that $y_B = 1$ is the unique best response to $y_A = 0$. When $\beta < \beta^{\text{rcv}}$, Lemma 6 implies that $y_B = 0$ is a strictly dominant strategy for B . Moreover, $\alpha > \alpha^{\text{rcv}}$ implies that $y_A = -1$ is the unique best response to $y_B = 0$. \square

Together, these lemmas yield the Proposition. \square

Proof of Proposition 5. Notice that

$$\begin{aligned} \alpha^{\text{rcv}} - \alpha^{\text{plu}} &= \frac{(1 - \beta)F(0) + \beta F(\theta - 1)}{F(\theta) + F(0)} - \frac{1 - \beta}{2} \\ &\propto 2(1 - \beta)F(0) + 2\beta F(\theta - 1) - (1 - \beta)(F(\theta) + F(0)) \\ &= (1 - \beta)F(0) + 2\beta F(\theta - 1) - (1 - \beta)F(\theta) \\ &= \beta[F(\theta) - F(0) + 2F(\theta - 1)] - F(\theta) + F(0) \end{aligned}$$

which yields

$$\hat{\beta} = \frac{F(\theta) - F(0)}{F(\theta) - F(0) + 2F(\theta - 1)} = \frac{\phi\theta}{\phi(3\theta - 2) + 2F(0)}$$

By inspection, $\hat{\beta}$ decreases in $2F(0) = 1 - 2\phi\bar{\rho}$, and thus it increases in $\bar{\rho}$. Differentiating with respect to θ yields

$$\frac{\partial \hat{\beta}}{\partial \theta} = \frac{2F(0) - 2\phi}{(\phi(3\theta - 2) + 2F(0))^2} = \frac{2F(-1)}{(\phi(3\theta - 2) + 2F(0))^2} > 0.$$

This completes the proof. \square

Proof of Proposition 6. Follows from the fact that β^{rcv} can be rewritten as

$$\max \left\{ \frac{(1-2\alpha)F(0)}{\phi(1-\phi\theta)} - \alpha \frac{\phi\theta}{1-\phi\theta}, \beta^{\text{plu}}, \frac{(\gamma-2\alpha)F(0)}{2\phi} + \frac{1+\alpha+\gamma\theta}{2} \right\} \geq \beta^{\text{plu}}.$$

This completes the proof. \square

Proof of Proposition 7. Realized turnout equals

$$\begin{aligned} T(y_A, y_B, \tau) &= \alpha F(\theta - 1 - y_A) + \beta F(\theta - 1 + y_B) + \gamma F(\theta) \\ &\quad + (1 - \alpha - \beta) \left\{ F(y_A + \tau) \mathbb{I}\{\tau \geq -(y_A + y_B)/2\} + F(-y_B - \tau) \mathbb{I}\{\tau < -(y_A + y_B)/2\} \right\} \end{aligned}$$

Expected turnout then equals

$$\begin{aligned} T^e(y_A, y_B) &= \alpha F(\theta - 1 - y_A) + \beta F(\theta - 1 + y_B) + \gamma F(\theta) + (1 - \alpha - \beta)F(0) \\ &\quad + \phi(1 - \alpha - \beta) \left\{ \int_{-(y_A+y_B)/2}^{\theta} (y_A + z) dG(z) - \int_{-\theta}^{-(y_A+y_B)/2} (y_B + z) dG(z) \right\} \\ &= (\alpha + \beta)\phi(\theta - 1) + \gamma\phi\theta + (1 + \gamma)F(0) - \phi\alpha y_A + \phi\beta y_B \\ &\quad + \phi(1 - \alpha - \beta) \left\{ \begin{aligned} &(1 - G(-(y_A + y_B)/2))y_A - G(-(y_A + y_B)/2)y_B \\ &+ \int_{-(y_A+y_B)/2}^{\theta} z dG(z) - \int_{-\theta}^{-(y_A+y_B)/2} z dG(z) \end{aligned} \right\} \\ &= T_0 - \phi\alpha y_A + \phi\beta y_B + \phi(1 - \alpha - \beta) \left\{ \begin{aligned} &G((y_A + y_B)/2)y_A - G(-(y_A + y_B)/2)y_B \\ &+ \int_{-(y_A+y_B)/2}^{\theta} z dG(z) + \int_{(y_A+y_B)/2}^{\theta} z dG(z) \end{aligned} \right\} \\ &= T_0 - \phi\alpha y_A + \phi\beta y_B + \phi(1 - \alpha - \beta) \left\{ \begin{aligned} &G((y_A + y_B)/2)y_A - G(-(y_A + y_B)/2)y_B \\ &+ \mathbb{E}\{\tau | \tau \geq 0\} - 2 \int_0^{|y_A+y_B|/2} z dG(z) \end{aligned} \right\} \end{aligned}$$

where $T_0 = (\alpha + \beta)\phi(\theta - 1) + \gamma\phi\theta + (1 + \gamma)F(0)$ and the last two lines follow from symmetry of $G(\cdot)$ around zero. Hence, we have

$$\begin{aligned} T^e(0, 0) &= T_0 + \phi(1 - \alpha - \beta) \mathbb{E}\{\tau | \tau \geq 0\} \\ T^e(-1, 0) - T^e(0, 0) &= \phi\alpha + \phi(1 - \alpha - \beta) \left\{ -G\left(-\frac{1}{2}\right) - 2 \int_0^{1/2} z dG(z) \right\} \end{aligned}$$

$$\begin{aligned}
&= \phi\alpha - \phi(1 - \alpha - \beta)G\left(\frac{1}{2}\right) \\
T^e(0, 1) - T^e(0, 0) &= \phi\alpha + \phi(1 - \alpha - \beta) \left\{ -G\left(-\frac{1}{2}\right) - 2 \int_0^{1/2} z dG(z) \right\} \\
&= \phi\beta - \phi(1 - \alpha - \beta)G\left(\frac{1}{2}\right) \\
T^e(-1, 1) - T^e(0, 0) &= \phi \{ \alpha + \beta - (1 - \alpha - \beta) \}
\end{aligned}$$

First, notice that whenever $\alpha + \beta > 1 - \alpha - \beta$, $T^e(-1, 1) > T^e(0, 0)$.

Second, notice that $T^e(-1, 1) - T^e(-1, 0) = \phi\alpha - \phi(1 - \alpha - \beta)(1 - G(1/2))$.

Hence, whenever $\beta > \beta^{plu}$, (i) $(-1, 1)$ is the turnout-maximizing profile, (ii) $(-1, 1)$ is the equilibrium under plurality but not under RCV. Hence, plurality dominates RCV in terms of turnout.

Conversely, when $\beta < \beta^{plu}$ (which implies $\beta < \beta^{rcv}$) and $\alpha \in [\max\{\alpha^{rcv}, (1 - \alpha - \beta)G(\frac{1}{2})\}, \alpha^{plu}]$, then RCV (asymmetric equilibrium) dominates plurality (centrist equilibrium). \square

Proof of Proposition 8. Suppose $\tau_{-1,0}^{AB} \geq \max\{\tau_{0,0}^{AB}, 0\}$, which implies that the unique equilibrium under RCV is $y_A = y_B = 0$. Since $\tau_{-1,0}^{AB} < \tau_{0,0}^{AB}$, we conclude that A wins more first preferences than B if and only if $\tau_0 \geq 0$. We have $\tau_0^A > 0$ if and only if

$$\gamma > \frac{\alpha F(\theta - 1) + \beta F(-\theta - 1) + (1 - \alpha - \beta)F(0)}{F(\theta)} \equiv \gamma_{0,0}^I(\theta),$$

in which case A 's total first and second preferences when $\tau = \tau_0^A$ are, by construction, equal to C 's total first and second preferences $\gamma F(\theta)$, and so A receives first and second preferences from fewer than a majority of all ballots cast when $\tau = \tau_0^A$ if and only if

$$\frac{\gamma F(\theta)}{2\gamma F(\theta) + \beta(F(\theta - 1) - F(-\theta - 1))} < \frac{1}{2},$$

which is true. The reason is that a positive measure of B 's core supporters with reservation utilities $-\theta - 1 < \rho < \theta - 1$ rank B first, but do not express a preference for A . Thus, if $\gamma_{0,0}^I(\theta)$, the claim holds, and it is immediate that $\gamma_{0,0}^I(\theta)$ decreases in θ , since $\gamma > \alpha$.

Suppose, however, $\gamma < \gamma_{0,0}^I(\theta)$. Then, whenever A defeats B , she also defeats C . At threshold $\tau_0 = 0$, A receives first and second preferences from fewer than a majority of all ballots cast if and only if

$$\frac{\alpha F(\theta - 1) + (1 - \alpha - \beta)F(0) + \beta F(-\theta - 1)}{\alpha F(\theta - 1) + (1 - \alpha - \beta)F(0) + \beta F(\theta - 1) + \gamma F(\theta)} < \frac{1}{2},$$

which is equivalent to

$$\gamma > \frac{\alpha F(\theta - 1) + (1 - \alpha - \beta)F(0) + \beta(2F(-\theta - 1) - F(\theta - 1))}{F(\theta)} \equiv \gamma_{0,0}^{II}(\theta).$$

It is immediate that $\gamma_{0,0}^{II}(\theta)$ decreases in θ . We conclude that if and only if $\gamma > \min\{\gamma_{0,0}^I(\theta), \gamma_{0,0}^{II}(\theta)\}$, A wins the election and receives first and second preferences from fewer than a majority of all ballots cast with positive probability.

By similar reasoning, if $\tau_0^B < 0$, which is equivalent to

$$\gamma > \frac{\alpha F(-\theta - 1) + \beta F(\theta - 1) + (1 - \alpha - \beta)F(0)}{F(\theta)} \equiv \gamma_{0,0}^{III}(\theta),$$

then B receives first and second preferences from fewer than a majority of all ballots cast when $\tau = \tau_0^B$ if and only if

$$\frac{\gamma F(\theta)}{2\gamma F(\theta) + \alpha(F(\theta - 1) - F(-\theta - 1))} < \frac{1}{2},$$

which is true. If, instead, $\tau_0^B > 0$, then B wins the election if and only if $\tau_0 < 0$; at that threshold (and so, for $\tau_0 < 0$ sufficiently close to zero), her first and second preferences are fewer than a majority of all ballots cast if

$$\frac{\beta F(\theta - 1) + (1 - \alpha - \beta)F(0) + \alpha F(-\theta - 1)}{\beta F(\theta - 1) + (1 - \alpha - \beta)F(0) + \alpha F(\theta - 1) + \gamma F(\theta)} < \frac{1}{2},$$

which is equivalent to

$$\gamma > \frac{\beta F(\theta - 1) + (1 - \alpha - \beta)F(0) + \alpha(2F(-\theta - 1) - F(\theta - 1))}{F(\theta)} \equiv \gamma_{0,0}^{IV}(\theta).$$

We conclude that if and only if $\gamma > \min\{\gamma_{0,0}^{III}(\theta), \gamma_{0,0}^{IV}(\theta)\}$, B wins the election and receives first and second preferences from fewer than a majority of all ballots cast with positive probability. So, if $\gamma > \min\{\gamma_{0,0}^I(\theta), \gamma_{0,0}^{II}(\theta), \gamma_{0,0}^{III}(\theta), \gamma_{0,0}^{IV}(\theta)\}$, the result holds whenever $\tau_{-1,0}^{AB} \geq 0$. Analysis for the remaining parameters is similar, and omitted. \square